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District Administration and Zilla Panchayath

and

Deputy Director, Department of School Education and Literacy

Kolar District, Kolar

2022-23

10th STANDARD

MATHEMATICS

WORK SHEETS

KEY ANSWERS

UNIT 1: ARITHMETIC PROGRESSION

I)

1) (A) $a_n=a+(n-1)\,d$
2) (C) 25
3) (C) 10,7,4,1, ------
4) (B) 10
5) (A) 210

II)

1) An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

2) $S_n = \frac{n}{2}[2a + (n-1)d)$ (3) $10^{th} term = m + 9p$

4) $a, a+d, a+2d, a+3d \dots\dots\dots..a+(n-1)d$

III)

1) $a = 2, d = 3, n = 12, a_{12} = ?$

$a_n = a + (n-1)d$

$= 2 + (12-1)3$

$= 2 + 33$

$\therefore a_n = 35$

2) $a = 3, d = 2, n = 20, S_{20} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2(3) + (20-1)2]$

$= 10(6 + 38)$

$= 10 \text{ x } 44$

$\therefore S_n = 440$

3) $a = 4, d = 5, n = ?, a_n = 154$

$a_n = a + (n-1)d$

$154 = 4 + (n-1)5$

$154 = 4 + 5n - 5$

$154 = 5n - 1$

$154 + 1 = 5n$

$n = \frac{155}{5}$

$n = 31$

$\therefore 154\ is\ a\ term\ of\ the\ given\ A, P.$

4) Given, $a_8 = 50\ \ and\ d = 6\ a = ?$

$=> a + 7d = 50$

$a + 7\,(6) = 50$

$a = 50 - 42$

$\therefore a = 8$

5) Given, x, $2x + p$ and $3x + 6$ are in A.P.

$=> 2x + p = \frac{x + 3x + 6}{2}$

$2(2x + p) = 4x + 6$

$4x + 2p = 4x + 6$

$2p = 6$

$\therefore p = 3$

6) $l = 101, d = 5, n = 15, a_{15} = ?$

$a_n = l - (n-1)d$

$= 101 - (15-1)5$

$= 101 - 70$

$\therefore a_{15} = 31$

$\therefore$ 15^{th} term from the last is 31

IV)

1) Data: $a_3 = 3$, $a_5 = -11$, $a_{50} = ?$

Solution: $\mathbf{a_n = a + (n-1)\,d}$

$a_3 = 3$ $\qquad$ $a_5 = -11$

a + 2d = 3-------(1) $\qquad$ a + 4d = -11-------(2)

(2) – (1)

a + 4d = -11

a + 2d = 3

\- \- \-

2d = - 14

d = -7

substitute d = -7 in (1) you get a = 17

$a_{50} = a + 49d$

$= 17 + 49(-7)$

$a_{50} = -326$

2) Solution: 4+8+12+16+------------+200

a= 4, d=4, a_n= 200, S_n=?

$\mathbf{a + (n-1)\,d = a_n}$

4 + (n-1) (4) = 200

n= 50

$S_n = \frac{n}{2}(a + an)$

$= \frac{50}{2}(4 + 200)$

= 5100

3) Data: $a_7 = 4a_2$, $a_{12} = 2 + 3a_4$, Ap=?

Solution: $\mathbf{a_n = a + (n-1)\, d}$

$a_7 = 4a_2$ $\quad\quad$ $a_{12} = 2 + 3a_4$

$a + 6d = 4(a + d)$ $\quad\quad$ $a + 11d = 2 + 3(a + 3d)$

$a + 6d = 4a + 4d$ $\quad\quad$ $a + 11d = 2 + 3a + 9d$

$3a = 2d$-------(1) $\quad\quad$ $2a - 2d = -2$-----(2)

Substitute (1) in (2) we get

$2a - 3a = -2$

$a = 2$

Substitute a=2 in (1) you get d= 3.

Therefore, given A.P. is 2, 5, 8, 11…………

4) Data: 1+2+3+4+………………+n=120

Sum of first n natural numbers = 120

$S_n=120$

$\mathbf{S_n = \frac{n(n+1)}{2}}$

$120=\frac{n(n+1)}{2}$

$240=n(n + 1)$

$15 \times 16=n(n + 1)$

$n = 15$

15 rows can be completed.

V)

1) $a_4 + a_8 = 24 \Rightarrow a + 3d + a + 7d = 24$

$2a + 10d = 24$ ----------------(1)

$a_6 + a_{10} = 44 \Rightarrow a + 5d + a + 9d = 44$

$2a + 14d = 44$ ---------------- (2)

Solving (1) and (2)

$$\begin{array}{rcl} 2a + 10d &=& 24 \\ 2a + 14d &=& 44 \\ - \quad - & & - \\ \hline -4d &=& -20 \end{array}$$

$d = 5$

Consider $a + 10d = 24$

$2a + 10(5) = 24$

$2a = 24 - 50$

$\therefore a = -13$

$\therefore \boldsymbol{a_1 = -13, a_2 = -8 \text{ and } a_3 = -3}$

2) Given l, b, h are in A.P.

Let, $a - d = l,$

$b = a$ *and* $h = a + d$

Given, $l + b + h = 15$

=> $a - d + a + a + d = 15$

$3a = 15$

$\therefore a = 5$

$Volume\ of\ the\ cuboid\ =\ lbh$

$80 = (a - d)(a)(a + d)$

$80 = (5 - d)(5)(5 + d)$

$16 = 25 - d^2$

$d^2 = 25 - 16$

$d = \sqrt{9}$ => $d = 3$

$\therefore l = 2\ units,\ \ b = 5\ units\ \ and\ h = 8\ units$

3) Here, $n = 10, d = -30.$

Let the amounts of the prizes be

$a, a - 30, a - 60, \ldots\ldots.., a - 270$

$a + a - 30 + a - 60 + \ldots\ldots + a - 270 = 2650$

$a = a,\ l = a - 270,$ $S_n = 2650$ $\quad n = 10$

$S_n = \frac{n}{2}[a + l]$

$S_{10} = \frac{10}{2}[a + a - 270]$

$2650 = 5(2a - 270)$

$2a - 270 = 530 \quad => a = 400$

Value of each prize is 400 370, 340, 310, 280, 250, 220, 190, 160, 130.

UNIT-2: TRIANGLES

I. Multiple choice questions:

1. B) 16: 81
2. A) 5cm
3. D) $\frac{AE}{EC}$
4. C) 6cm
5. C) 6, 8, 10

II. One mark questions:

1. **Thales' theorem(Basic proportionality theorem):**
   If a line drawn parallel to one side of a triangle to intersect the other two sides in two distinct points, then the other two sides are divided in the same ratio.

2. **Pythagoras' theorem**: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

3. $BC^2 = AC^2 - AB^2$

4. $\frac{KN}{KL} = \frac{KP}{KM}$ Or $\frac{KN}{KL} = \frac{NP}{LM}$

III. Two marks questions:

1. Given, Area of $\triangle PQR$ = 64sqcm, Area of $\triangle ABC$ = 144sqcm, QR= 8cm, BC=?
   Given $\triangle PQR \sim \triangle ABC$

$$\therefore \frac{area\ of\ \triangle PQR}{area\ of\ \triangle ABC} = \frac{QR^2}{BC^2}$$

$$\frac{64}{144} = \left(\frac{QR}{BC}\right)^2$$

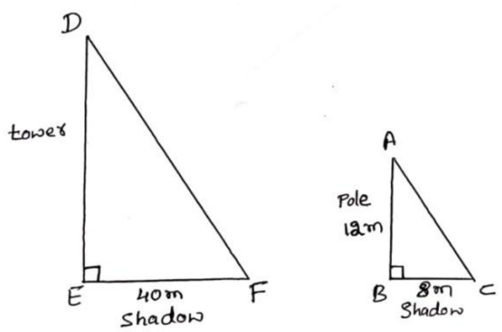
$$\therefore \sqrt{\frac{64}{144}} = \frac{QR}{BC}$$

$$\frac{8}{12} = \frac{8}{BC}$$

$$BC = \frac{12X8}{8}$$

BC=12cm

2. Given: ABC is an isosceles triangle.
   $\therefore AB = BC$
   In $\triangle ABC$, $\angle B = 90^0$
   From Pythagoras theorem
   $AC^2 = AB^2 + BC^2$
   $= AB^2 + AB^2$ ($\because AB = BC$)
   $AC^2 = 2AB^2$



3. Length of the vertical pole = AB = 12m
   Length of the shadow casts by the pole = BC = 8m
   Length of the shadow casts by the tower = EF = 40m
   Let the height of the tower = h m

In ΔABC and ΔDEF

$\angle B = \angle E = 90^0$

$\angle C = \angle F$ [The angles made by sun at the same time]

$\therefore \Delta ABC \sim \Delta DEF$ [By AA-criterion of similarity]

$\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{12}{h} = \frac{8}{40}$

$\frac{12\times40}{8} = h$

$h = 60$

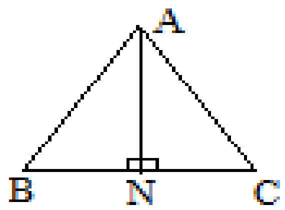
$\therefore$ **Height of the tower = 60m.**

IV. Three marks questions:

1. ABC is an equilateral triangle.

$\therefore$AB = BC = AC and

$\angle ABC = \angle BAC = \angle ACB = 60^0$



Since AN $\perp$ BC

$\therefore \angle ANB = \angle ANC = 90^0$ and

BN = NC

In ΔANB, $\angle ANB = 90^0$

$\therefore AB^2 = BN^2 + AN^2$

$AB^2 = (\frac{1}{2} BC)^2 + AN^2$

$= \frac{1}{4} BC^2 + AN^2$

$AB^2 = \frac{BC^2 + 4AN^2}{4}$

$4AB^2 = BC^2 + 4AN^2$

$4AB^2 = AB^2 + 4AN^2$ ($\because$BC = AB)

$4AB^2 - AB^2 = 4AN^2$

$\mathbf{3AB^2 = 4AN^2}$

2. *In* ΔAOB *and* ΔCOD we have,

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles, AB||DC]

$\Delta AOB \sim \Delta COD$ [By AA-criterion]

$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2}$

$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2DC)^2}{(DC)^2} = \frac{4}{1}$

$\therefore \boldsymbol{ar(\Delta AOB) : ar(\Delta COD) = 4 : 1}$

V. Four/ Five mark questions :

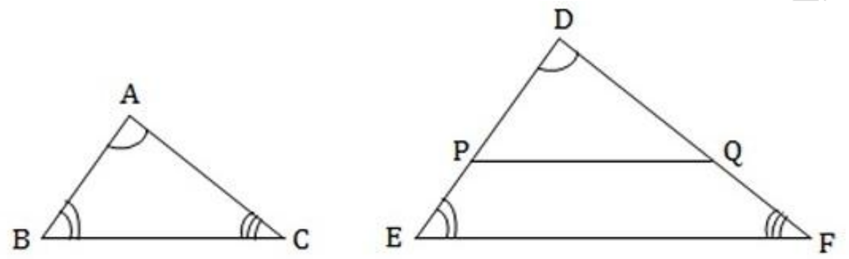
1. Prove that "If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (proportion) and hence the two triangles are similar".

Data: $In\ \Delta ABC\ and\ \Delta DEF$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$



To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

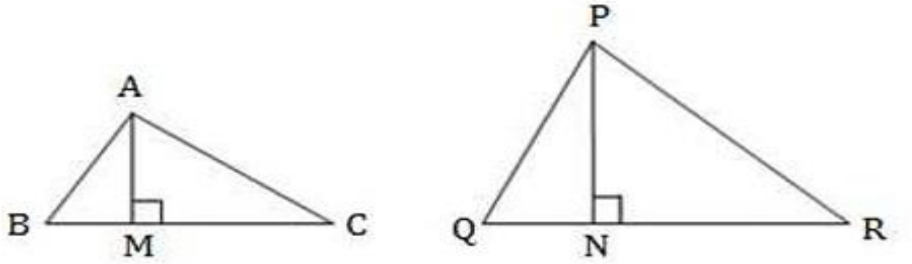
Construction: Mark points P and Q on DE and DF such that DP=AB and DQ=AC. Join PQ.

Proof: $In\ \Delta ABC\ and\ \Delta DPQ$

$\angle A = \angle D$	[Data]
AB=DP	[Construction]
AC=DQ	[Construction]
$\therefore \Delta ABC \cong \Delta DPQ$	[SAS postulate]
BC=PQ	[By CPCT] -----(1)
$\angle B = \angle P$	[By CPCT]
$\angle B = \angle E$	[Data]
$\angle P = \angle E$	[Axiom 1]
$PQ \parallel EF$	
$\frac{DP}{DE} = \frac{PQ}{EF} = \frac{DQ}{DF}$	[Corollary of BPT]
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	[From (1) and construction]

$\therefore$ **Hence the proof.**

2. Prove that "The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides".



Data : $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction : Draw $AM \perp BC$ and $PN \perp QR$.

Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ ($\because$ Area of $\Delta = \frac{1}{2} \times$ base $\times$ height)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AM}{QR \times PN}$ ----------> (1)

In ΔABM and ΔPQN

$\angle B = \angle Q$ ($\because \Delta ABC \sim \Delta PQR$)

$\angle M = \angle N = 90^\circ$ ($\because$Construction)

$\therefore \Delta ABM \sim \Delta PQN$ ($\because$AA Similarity criterion)

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ --------> (2)

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ------> (3) ($\because$Data)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ ($\because$substituting eqs.(2) and (3) in (1))

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$

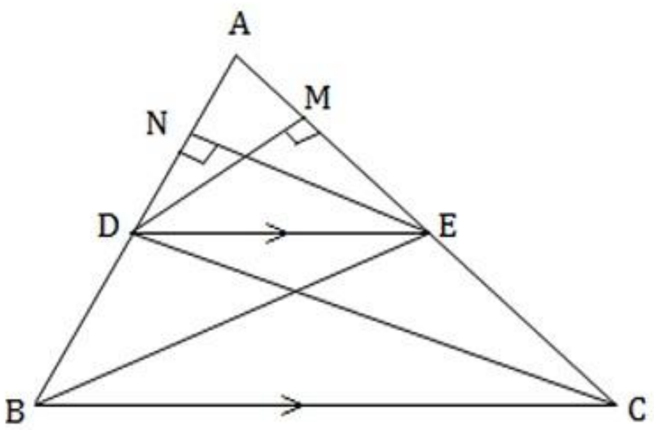
Now from eq.(3)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Hence the proof.

3. State and prove the Basic proportionality (Thales') theorem.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Data : In ΔABC $DE \parallel BC$.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw $DM \perp AC$ and $EN \perp AB$. Join BE and CD.

Proof :

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad \text{---------->} (1)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{AE}{EC} \quad \text{---------->} (2)$$

But ΔBDE and ΔCED are standing on the same base DE and between $DE \parallel BC$.
$ar(\Delta BDE) = ar(\Delta CED)$ ----------> (3)

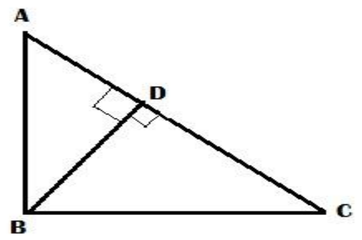
$\therefore$ from equations (1), (2) and (3)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence the proof.

4. . State and prove the Pythagoras theorem.

" In a right angled triangle , the square on the hypotenuse is equal to the sum of the squares on other two sides " .



Data : ΔABC is a right triangle and $\angle B = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In ΔADB and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle A = \angle A$ ($\because$ Common angle)

$\Delta ADB \sim \Delta ABC$ ($\because$ AAA Similarity Criterion)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ ($\because$ Proportional sides)

$AC. AD = AB^2$ ------> (1)

Similarly

In ΔBDC and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle C = \angle C$ ($\because$ Common angle)

$\Delta BDC \sim \Delta ABC$ ($\because$AAA Similarity Criterion)

$\therefore \frac{DC}{BC} = \frac{BC}{AC}$ ($\because$ Proportional sides)

$AC. DC = BC^2$ ------> (2)

$AC . AD + AC. DC = AB^2 + BC^2$ [$\because$ By adding (1) and (2)]

$AC (AD + DC) = AB^2 + BC^2$

$AC \times AC = AB^2 + BC^2$ ($\because$ from fig. $AD + DC = AC$)

$AC^2 = AB^2 + BC^2$

Hence the proof.

UNIT-3: PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Multiple Choice Questions

1 (B) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

2 (C) Exactly one solution

3 (A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

4 (D) coincident lines

5 (B) Consistent

One Mark Questions

1 $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where a_1, b_1, c_1, a_2, b_2 and c_2 are all real numbers

2 Infinitely many solutions.

3 $x = 2$ and $y = 1$

Two marks questions:

1

$$\begin{array}{c} x+y=8 \\ 2x-y=7 \\ \hline 3x=15 \\ x=5 \end{array} \quad \text{(addition)}$$

Substituting the value of x in $x + y = 8$

$5 + y = 8$

$y = 3$

$\therefore \boldsymbol{x = 5 \text{ and } y = 3}$

2

$x + y = 5$ ----- (1)

$2x + 3y = 12$ ----- (2)

Multiplying the equation (1) by 2 we get

$2x + 2y = 10$ ----- (3)

Solving equation (2) and (3)

$$\begin{array}{c} 2x+3y=12 \\ 2x+2y=10 \\ \hline y=2 \end{array} \quad \text{(subtraction)}$$

Substitute the value of y in $x + y = 5$,

$x + 2 = 5$

$x = 3$

$\therefore \boldsymbol{x = 3 \text{ and } y = 2}$

Three marks questions:

1 Let the two numbers be x and y.

According to the data

$x + y = 50$ ------ (1)

$x - y = 22$ ------ (2)

Solving (1) and (2)

$$\begin{array}{c} x+y=50 \\ \underline{x-y=22} \\ 2x=72 \\ x=36 \end{array} \quad \text{(addition)}$$

By Substituting the value of x in (1) we get

$x + y = 50$

$36 + y = 50$

$y = 14$

$\therefore$ $\boldsymbol{The\ two\ numbers\ are\ 36\ and\ 14}$

2 Let the age of son be *'x'* years and

the age of father be *'y'* years

$2x + y = 56$ ------ (1)

$x + 2y = 82$ ------ (2)

Multiply the equation (2) by 2 we get

$2x + 4y = 164$ ---- (3)

Solving (1) and (3)

$$\begin{array}{c} 2x+4y=164 \\ \underline{2x+y=56} \\ 3y=108 \\ y=36 \end{array}$$

By substituting the value of y in (1) we get x=10

$\therefore$ **The age of the son and the age of father are 10 years and 36 years respectively**.

Four marks questions:

1

$x + y = 5$

x	0	5
y	5	0

$x - y = 1$

x	0	1
y	-1	0

If x = 0

x + y = 5

0 + y = 5

y = 5

If x = 0

x − y = 1

0 − y = 1

y = −1

If y = 0

x + y = 5

x + 0 = 5

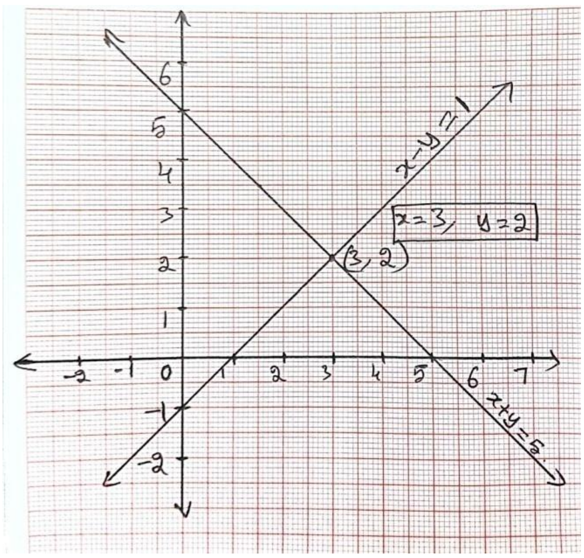
x = 5

If y = 0

x − y = 1

x − 0 = 1

x = 1

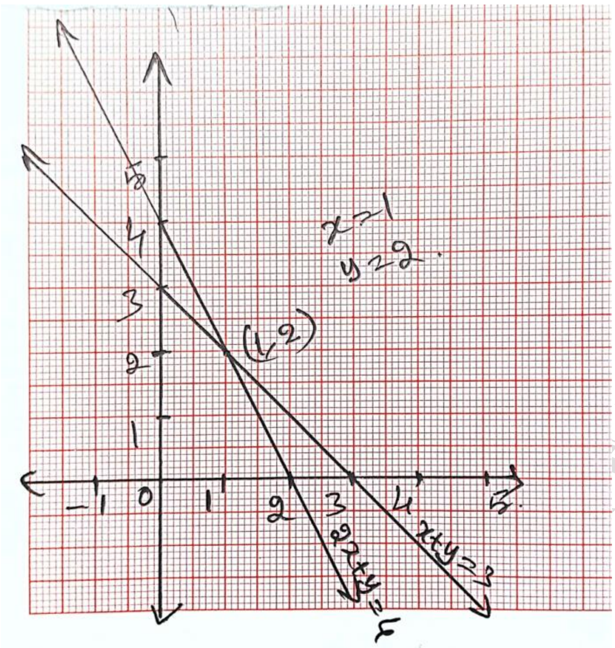


$\therefore$ $\boldsymbol{x = 3\ and\ y = 2}$

2 $2x + y = 4$ $x + y = 3$

x	0	2
y	4	0

x	0	3
y	3	0



If x = 0
$2x + y = 4$
$0 + y = 4$
$y = 4$

If y = 0
$2x + y = 4$
$2x + 0 = 4$
$x = 2$

If x = 0
$x + y = 3$
$0 + y = 3$
$y = 3$

If y = 0
$x + y = 3$
$x + 0 = 3$
$x = 3$

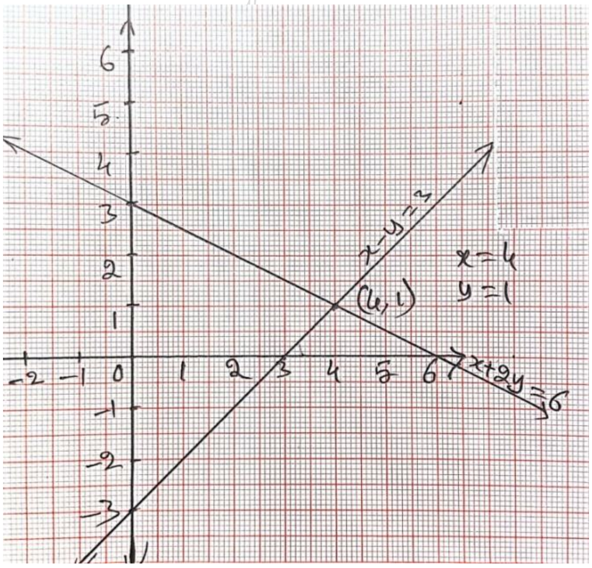
$\therefore \boldsymbol{x = 1 \text{ and } y = 2}$

3

$x + 2y = 6$ *and* $x - y = 3$

x	0	6
y	3	0

x	0	3
y	-3	0



If x = 0
$x + 2y = 6$
$0 + 2y = 6$
$y = 3$

If y = 0
$x + 2y = 6$
$x + 0 = 6$
$x = 6$

If x = 0
$x - y = 3$
$0 - y = 3$
$y = -3$

If y = 0
$x - y = 3$
$x - 0 = 3$
$x = 3$

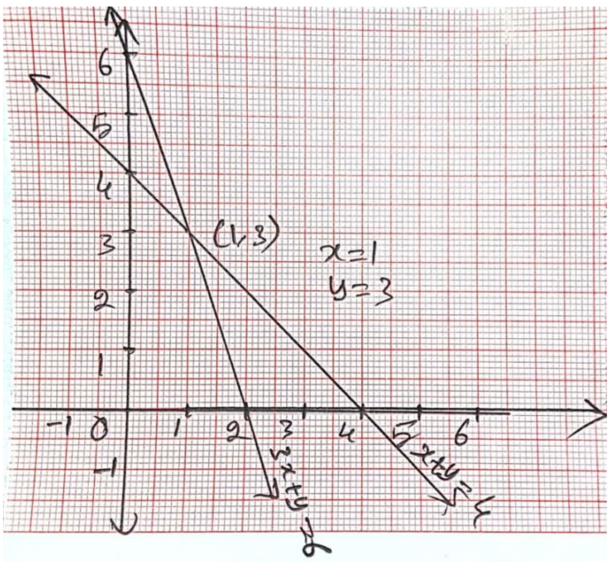
$\therefore \boldsymbol{x = 4 \text{ and } y = 1}$

4

$3x + y = 6$ *and* $x + y = 4$

x	0	2
y	6	0

x	0	4
y	4	0



If x = 0
$3x + y = 6$
$0 + y = 6$
$y = 6$

If y = 0
$3x + y = 6$
$3x + 0 = 6$
$x = 2$

If x = 0
$x + y = 4$
$0 + y = 4$
$y = 4$

If y = 0
$x + y = 4$
$x + 0 = 4$
$x = 4$

$\therefore \boldsymbol{x = 1 \text{ and } y = 3}$

UNIT-4: CIRCLES

I) Multiple Choice Questions

1 **B) parallel to each other**

2 **C) secant**

3 **A) 2**

4 **A) 70^o**

5 **A) 50^0**

II) One Mark Questions.

1 The measure of the angle between radius and tangent at the point of contact is 90^0

2 A straight line which intersects a circle at two distinct points is called the secant of a circle

3 A straight line which touches a circle at only one point is called the tangent of a circle

III) Two Marks Questions

1 In adjoining fig OA=6cm, AP-8cm, $\angle A=90^0$

By Pythagoras theorem

$OP^2 = OA^2 + AP^2$

$= (6)^2 + (8)^2$

$= 36 + 64$

$= 100$

$OP = \pm 10$

d =10 cm

2 In adjoining fig

OR=3cm,OQ=4cm,$\angle R=90^0$

By Pythagoras theorem

$OQ^2 = OR^2 + RQ^2$

$OQ^2 - OR^2 = RQ^2$

$(5)^2 - (3)^2 =$

$25 - 9 =$

$16 = RQ^2$

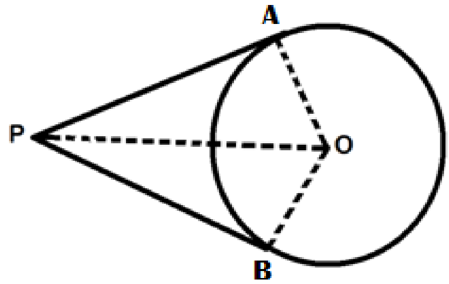
$RQ = \pm 4$

PQ = 2RQ =2×4 =8 cm

IV) Three Marks Questions

1. Prove that "the length of tangents drawn from an external point to a circle is equal."

Given : 'O' is the centre of the circle, 'P' is an external point. AP and BP are the tangents



To Prove : AP = BP

Construction : Join OA, OB and OP.

Proof :

In ΔOQP and ΔORP

$\angle$OAP = $\angle$OBP = 90^0 [Theorem 4.1]

OP = OP [Common side]

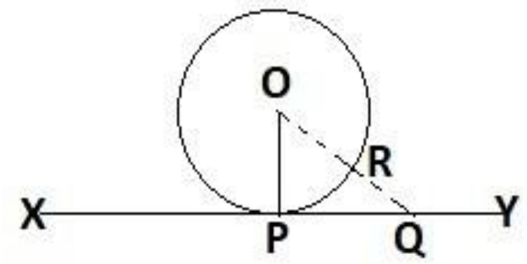
OA = OB [Radii of same circle]

$\Delta OAP \cong \Delta OBP$ [RHS Postulate]

AP=BP [CPCT]

2. Prove that "the tangent at any point of a circle is perpendicular to the radius through the point of contact."

Given : XY is the tangent at P to the circle with centre



To Prove : OP ⊥ XY

Construction : Mark Any point 'Q' on XY, join OQ and it cuts the circle at R

Proof : OR < OQ

OR = OP (Radii of the same circle)

∴OP < OQ

This holds good for all the points on XY

∴ OP is the least distance from the centre to the tangent.

=>OP ⊥XY

UNIT 5. AREAS RELATED TO CIRCLES

I) Multiple Choice Questions

1. A. $2\pi cm$
2. C. 8:9
3. B.$44cm$

II) One Mark Questions

1. $\frac{\theta}{360^0}$ x πr^2
2. The area bounded by two radii and the corresponding arc of a circle is calledthe Sector.
3. A segment is a region covered by a chord and a corresponding arc.

III) Two Marks Questions

1. Length of the arc=$\frac{\theta}{360^o}$ x $2\pi r$

   =$\frac{90^o}{360^o}$ x 2 x $\frac{22}{7}$x 35

   ∴ **Length of the arc = 55cm**

2. Radius of each quadrant = $\frac{28}{2}$ = 14 cm

   Area of the shaded region = Area of the square – Area of 4 Quadrants.

   Area of the shaded region $= 28^2 - 4$ x$\frac{\pi r^2}{4}$

   $= 784 - 4$ x$\frac{22}{7}$x$\frac{14\text{x}14}{4}$

   $= 784 - 616$

   ∴**Area of the shaded region= $\mathbf{168cm^2}$**

3. Radius of the circle; r = $\frac{42}{2}$ = 21cm

   Ar(shaded region) = Ar(Square) – Ar(Circle)

   $= (side)^2 - \pi r^2$

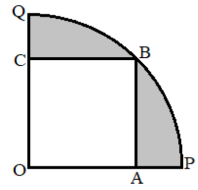
   $= 42^2 - \frac{22}{7}$x21x21

   $= 1764 - 1386$

   ∴**Area of the shaded region= $\mathbf{378cm^2}$**

IV) Three Marks Questions.

1. OABC is a square inscribed in a quadrant OPBQ. If OA = 20 cm. (use $\pi = 3.14$)



$$\text{Ar(Square)} = 20^2$$
$$= 400cm^2$$

Radius of the quadrant; r= OB

$r = OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{20^2 + 20^2}$

$r = 20\sqrt{2}\ cm$

Ar(Quadrant) = $\frac{\pi r^2}{4}$

$= \frac{3.14 \text{ x} \left(20\sqrt{2}\right)^2}{4}$

$= \frac{3.14\ x\ 400\ x\ 4}{4}$

Ar(Quadrant)= $628\ cm^2$

Ar(Shaded region)= $\boldsymbol{Ar(Quadrant) - Ar(Square)}$

$$= 628 - 400$$

$\therefore$ $\boldsymbol{Area\ of\ the\ shaded\ region = 228\ cm^2}$

2. Area of a sector of a circle of radius 14 cm is 154 cm^2. Find the length of the corresponding arc of the sector.

Given, $r = 14\ cm$ Area of sector $= 154\ cm^2$

$\frac{\theta}{360^o} \text{ x } \pi r^2 = 154$

$\frac{\theta}{360^o} \text{ x } \frac{22}{7} \text{ x } 14 \text{ x } 14 = 154$

$\frac{\theta}{360^o} \text{ x } 22 \text{ x } 2 \text{ x } 14 = 154$

$\theta = \frac{154 \text{ x } 360}{22 \text{ x } 2 \text{ x } 14} \Rightarrow \theta = 90^0$

Length of an arc= $\frac{\theta}{360^o} \text{ x } 2\pi r$

$$= \frac{90^o}{360^o} \text{ x } 2 \text{ x } \frac{22}{7} \text{ x } 14$$

$\therefore$**Length of the arc=** $\boldsymbol{22\ cm}$

UNIT 7: COORDINATE GEOMETRY

Multiple choice questions

1) (C) 5 units 2) (D) 5 units 3) (B) (6, 0) 4) (D) Forth quadrant

Very Short Answer questions (1 Mark)

1)$(0, 0)$

2) $(0, 5)$

3) $7\ units$

4) Zero

2 Marks

1) $(2, 3)$, $(-1, 7)$

(x_1, y_1), (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-1 - 2)^2 + (7 - 3)^2}$$

$$= \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$\mathbf{d = 5\ units}$$

2) $(2, 3) \rightarrow (x_1,\ y_1)$, $(4, 7) \rightarrow (x_2,\ y_2)$

Mid-point = $\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)$

Mid-point =$\left(\frac{2+4}{2},\ \frac{3+7}{2}\right)$

$$= \left(\frac{6}{2}, \frac{10}{2}\right)$$

$$\mathbf{Mid-point = (3, 5)}$$

3) $Point\ on\ x-axis(x, 0) \rightarrow (x,\ y)$,$(0, 3) \rightarrow (x_1,\ y_1)$, $(4, -1) \rightarrow (x_2,\ y_2)$

Section formula $(x,\ y)$ = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\ \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

y = $\frac{m_1y_2+m_2y_1}{m_1+m_2}$

0 = $\frac{m_1(-1)+m_2(3)}{m_1+m_2}$

$$0 = -m_1 + 3m_2$$

$$m_1 = 3m_2$$

$$\boldsymbol{m_1 : m_2 = 3 : 1}$$

4) $(5, p)$ ➔$(x_1,\ y_1)$,   (2, 0) ➔$(x_2,\ y_2)$, $d = 5\ units.$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(2-5)^2 + (0-p)^2}$ [Squaring both sides]

$25 = (-3)^2 + p^2$

$25 = 9 + p^2$

$16 = p^2$

$p = \pm 4$

3 Mark

1) $A\ (0, 2),\ is\ equidistant\ from\ (3, m)\ and\ (m, 3)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$Distance\ from\ A(0,2)\ to\ (3, m) = Distance\ from\ A(0,2) to\ (m, 3)$

$\sqrt{(3-0)^2 + (m-2)^2} = \sqrt{(m-0)^2 + (3-2)^2}$ [Squaring both sides]

$3^2 + m^2 - 2(m)(2) + 2^2 = m^2 + 1^2$

$9 - 4m + 4 = 1$

$9 + 4 - 1 = 4m$

$\mathbf{m = 3}$

2) Area of $\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

P (1, 2) ➔ (x_1, y_1),Q (3, 7) ➔ (x_2, y_2),R(5, 3) ➔ (x_3, y_3).

Area of $\Delta ABD = \frac{1}{2}[1(7-3) + 3(3-2) + 5(2-7)]$

$= \frac{1}{2}[1(4) + 3(1) + 5(-5)]$

$= \frac{1}{2}[4 + 3 - 25] = \frac{1}{2}[-18] = -9\ sq.units = 9\ sq.units.$

3) Let A $(0, 0)$ ➔$(x_1,\ y_1)$andB (3, 0) ➔$(x_2,\ y_2)$ then we need to find third vertex $C(x, y)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(3-0)^2 + (0-0)^2}$➔$AB = \sqrt{9}$➔$AB = 3\ units$

Since the triangle is equilateral, therefore all the sides are equal,

$AC = BC$

$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(x-3)^2 + (y-0)^2}$[Squaring both sides]

$x^2 + y^2 = x^2 - 2(x)(3) + 3^2 + y^2[(x-y)^2 = x^2 - 2xy + y^2]$

$0 = -6x + 9$ ➔$6x = 9$➔$x = \frac{9}{6}$➔$x = \frac{3}{2}$

$AC = AB$

$\sqrt{(x-0)^2 + (y-0)^2} = 3$ [Squaring both sides]

$x^2 + y^2 = 3$➔$\left(\frac{3}{2}\right)^2 + y^2 = 3$➔$y^2 = 3 - \frac{9}{4}$➔$y^2 = \frac{3}{4}$➔$y = \frac{\sqrt{3}}{2}$

∴ The third vertex is $C(x, y) = C\left(\frac{3}{2},\ \frac{\sqrt{3}}{2}\right)$

4/5 Mark

1) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$A(-4,-1) \rightarrow (x_1, y_1)$, $B(-2,-4) \rightarrow (x_2, y_2)$.

$AB = \sqrt{(-2-(-4))^2 + (-4-(-1))^2} = \sqrt{(-2+4)^2 + (-4+1)^2}$

$= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$ units.

$B(-2,-4) \rightarrow (x_1, y_1)$, $C(4,0) \rightarrow (x_2, y_2)$.

$BC = \sqrt{(4-(-2))^2 + (0-(-4))^2} = \sqrt{(4+2)^2 + (0+4)^2}$

$= \sqrt{6^2 + 4^2} = \sqrt{36+16} = \sqrt{52}$ units.

$C(4,0) \rightarrow (x_1, y_1)$, $D(2,3) \rightarrow (x_2, y_2)$.

$CD = \sqrt{(2-4))^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$ units.

$D(2,3) \rightarrow (x_1, y_1)$, $A(-4,-1) \rightarrow (x_2, y_2)$.

$DA = \sqrt{(-4-2))^2 + (-1-3))^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$ units.

$AB = CD \text{ and } BC = DA$, Opposite sides are equal.

$A(-4,-1) \rightarrow (x_1, y_1)$, $C(4,0) \rightarrow (x_2, y_2)$.

$AC = \sqrt{(4-(-4))^2 + (0-(-1))^2} = \sqrt{(4+4)^2 + (0+1)^2}$

$= \sqrt{8^2 + 1^2} = \sqrt{64+1} = \sqrt{65}$ units.

$B(-2,-4) \rightarrow (x_1, y_1)$, $D(2,3) \rightarrow (x_2, y_2)$.

$BD = \sqrt{(2-(-2))^2 + (3-(-4))^2} = \sqrt{(2+2)^2 + (3+4)^2}$

$= \sqrt{4^2 + 7^2} = \sqrt{16+49} = \sqrt{65}$ units.

AC = BD, Diagonals are equal.

∴ ABCD is a rectangle.

2) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$A\ (2, 5) \rightarrow (x_1, y_1)$, $B\ (2, 1) \rightarrow (x_2, y_2)$.

$AB^2 = (2 - 2)^2 + (1 - 5)^2 = (0)^2 + (-4)^2 = 0 + 16 = 16\ units.$

$B\ (2, 1) \rightarrow (x_1, y_1)$, $C\ (5, 1) \rightarrow (x_2, y_2)$.

$BC^2 = (5 - 2)^2 + (1 - 1)^2 = (3)^2 + (0)^2 = 9 + 0 = 9\ units.$

$A\ (2, 5) \rightarrow (x_1, y_1)$, $C\ (5, 1) \rightarrow (x_2, y_2)$.

$AC^2 = (5 - 2)^2 + (1 - 5)^2 = (3)^2 + (4)^2 = 9 + 16 = 25\ units.$

$AB^2 + BC^2 = AC^2 \rightarrow 16 + 9 = 25.$

$\therefore \Delta ABC$ is a right-angled triangle. [Converse of Pythagoras theorem]

The length of the median from the vertex C.

Mid-point = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow$ Mid-point of AB $\rightarrow \left(\frac{2+2}{2}, \frac{5+1}{2}\right) \rightarrow \left(\frac{4}{2}, \frac{6}{2}\right)$

$\rightarrow (2, 3) \rightarrow (x_1, y_1)$, $C\ (5, 1) \rightarrow (x_2, y_2)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$Length\ of\ the\ Median\ from\ vertex\ C = \sqrt{(5 - 2)^2 + (1 - 3)^2}$$

$$= \sqrt{(3)^2 + (-2)^2}$$

$$= \sqrt{9 + 4}$$

$$= \sqrt{13}\ units$$

UNIT 8: REAL NUMBERS

I) Multiple Choice Questions

1 C) $\frac{3}{5}$

2 B) 90

3 B) Composite number

4 A) $a = bq + r$

II) One Mark Questions

1. Given positive integers a and b ,there exists unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$
2. Every composite number can be expressed as a product of primes & this factorization is unique apart from order on which the prime factors occur
3. H.C.F of any two prime numbers is one
4. $90 = 2^1 \times 3^2 \times 5^1$

III) Two Marks Questions

1. Proof : let $5 - \sqrt{3}$ be a rational number.

$\rightarrow 5 - \sqrt{3} = \frac{p}{q}$ where $q \neq 0$, $p,q \in Z$

$5 - \frac{p}{q} = \sqrt{3}$

$\frac{5q - p}{q} = \sqrt{3}$

Rational number ≠ irrational number

LHS ≠ RHS

Our assumption is wrong

$\therefore 5 - \sqrt{3}$ is an irrational number

2. Proof : let $\sqrt{2} + 7$ be a rational number.

$\rightarrow \sqrt{2} + 7 = \frac{p}{q}$ where $q \neq 0$, $p, q \in Z$

$\sqrt{2} = \frac{p}{q} - 7$

$\sqrt{2} = \frac{p - 7q}{q}$

Irrational number ≠ Rational number

LHS ≠ RHS

Our assumption is wrong

$\therefore \sqrt{2} + 7$ is an irrational number

3. Prime factors of 401 $=2^2\times101$

Prime factors of 96 $=2^5\times3^1$

HCF(404,96) $=2^2=4$

V) Three Marks Questions.

1) Let $\sqrt{2}$ be a rational number.

$\rightarrow \sqrt{2}=\frac{p}{q}$ where $q\neq0$, p and q are co-prime

$q\sqrt{2}=p$ square on both sides

$2q^2=p^2$ ……………(1)

$\therefore p^2$ is divisible by 2 and also p is divisible by 2 ………(2)

$\rightarrow$ $p=2r$

Substitute $p=2r$ in (1)

$2q^2=(2r)^2$

$2q^2=4r^2$

$q^2=2r^2$

$\therefore q^2$ is divisible by 2 and also q is divisible by 2 ……..(3)

$\therefore$ from (2) and (3)

Both p and q have common factors.

i.e., p and q are not co-prime. It is contradictory to our assumption.

$\therefore \sqrt{2}$ is an irrational number.

2) let $\sqrt{3}$ be a rational number.

$\rightarrow \sqrt{3}=\frac{p}{q}$ where $q\neq0$, p and q are co-prime

$q\sqrt{3}=p$ square on both sides

$3q^2=p^2$ ……………(1)

$\therefore p^2$ is divisible by 3 and also p is divisible by 3 ………(2)

$\rightarrow$ $p=3r$

Substitute $p=3r$ in (1)

$3q^2=(3r)^2$

$3q^2=9r^2$

$q^2=3r^2$

$\therefore q^2$ is divisible by 3 and also q is divisible by 3 ……..(3)

$\therefore$ from (2) and (3)

p and q have common factors.

i.e., both p and q are not co-prime. It is contradictory to our assumption.

$\therefore \sqrt{3}$ is an irrational number.

UNIT 9: POLYNOMIALS

I) Multiple Choice Questions

1) **(C) 4** 2) **(D) 0** 3) **(B) 5** 4) **(B) 1** 5) **(A)** $\mathbf{\frac{2}{3}}$

6) **(A) 8** 7) **(D) 2** 8) **(C)** $-\mathbf{\frac{7}{5}}$ 9) **(A) 6**

II) One Mark Questions

1) $a = 1,\ b = 0,\ c = -9.$

Sum of zeroes = $-\frac{b}{a} = -\ \frac{0}{1}$

$\therefore$ Sum of zeroes $= 0$

2) $x^2 - 3 = 0$

$(x + \sqrt{3})(x - \sqrt{3}) = 0$

$\therefore x = -\sqrt{3}\ and\ x = +\sqrt{3}$

3) If $p(x)\ and\ g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find the polynomials $q(x)\ \ and\ r(x)$ such that,

$\boldsymbol{p(x) = g(x)}$ **x** $\boldsymbol{q(x) + r(x).}$ Where $r(x) = 0$ or degree of $r(x) <$ degree $g(x)$

III) Two Mark Questions

1) Let $\alpha = -1\ and\ \beta = 5$

$\alpha + \beta = 4$

$\alpha\beta = -5$

The quadratic polynomial is of the form

$x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore The\ polynomial\ is\ x^2 - 4x - 5$

2) Let $\alpha\ and\ \beta$ be the zeroes of the polynomial

Given,

$\alpha + \beta = 4$

$\alpha\beta = 1$

The quadratic polynomial is of the form

$x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore The\ polynomial\ is\ x^2 - 4x + 1$

3) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

$zero\ of\ x - 4\ \ is\ 4$ and

$and\ zero\ of\ x + 2\ is - 2$

$\therefore zeroes\ of\ x^2 - 2x - 15\ are\ 4\ and - 2$

4) We know, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$$\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$$
$$=\frac{c}{a}\left(-\frac{b}{a}\right)$$
$$\therefore \alpha^2\beta+\alpha\beta^2=-\frac{bc}{a^2}$$

5)

$$\begin{array}{r|l}
 & 3x-5 \\ \hline
x^2+2x+1 & 3x^3+x^2+2x+5 \\
 & 3x^3+6x^2+3x \\
 & (-)\quad(-)\quad(-) \\ \hline
 & -5x^2-x+5 \\
 & -5x^2-10x-5 \\
 & (+)\quad(+)\quad(+) \\ \hline
 & 9x+10
\end{array}$$

$\therefore Quotient = 3x-5$ *and*
$Remainder = 9x+10$

6)

$$\begin{array}{r|l}
 & x-4 \\ \hline
x+3 & x^2-x-12 \\
 & x^2+3x \\
 & (-)\quad(-) \\ \hline
 & -4x-12 \\
 & -4x-12 \\
 & (+)\quad(+) \\ \hline
 & 0
\end{array}$$

Here the remainder =0
$\therefore (x+3)$ is a factor of x^2-x-12.
(Since the divisor is a linear polynomial here, it also be solved using Remainder theorem.)

IV) Three Mark Questions

1)

$$\begin{array}{r|l}
 & x-2 \\ \hline
x^2-x+1 & x^3-3x^2+3x-5 \\
 & x^3-x^2+x \\
 & (-)\quad(+)\quad(-) \\ \hline
 & -2x^2+2x-5 \\
 & -2x^2+2x-2 \\
 & (+)\quad(-)\quad(+) \\ \hline
 & -3
\end{array}$$

Verification of Division algorithm:

$g(x)$ x $q(x) + r(x) = p(x)$

$$= (x^2 - x + 1)(x - 2) + (-3)$$
$$= x^3 - x^2 + x - 2x^2 + 2x - 2 - 3$$
$$= x^3 - 3x^2 + 3x - 5$$
$$= p(x)$$

2) Let $\alpha = 2 + \sqrt{3}$ and $\beta = 2 - \sqrt{3}$.

$\alpha + \beta = 2 + \sqrt{3} + 2 - \sqrt{3}$

$\therefore \alpha + \beta = 4$

$\alpha\beta = (2 + \sqrt{3})(2 - \sqrt{3})$

$= 2^2 - (\sqrt{3})^2$

$= 4 - 3$

$\therefore \alpha\beta = 1$

The quadratic polynomial is of the form

$x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore The\ polynomial\ is\ x^2 + 4x + 1$

3) Given, $q(x) = x - 2$ and $r(x) = (-2x + 4)$

We know, $g(x)$x $q(x) + r(x) = p(x)$

$g(x)$ x $(x - 2) + (-2x + 4) = x^3 - 3x^2 + x + 2$

$g(x)$ x $(x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$$\therefore g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & (-) \quad (+) \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & (+) \quad (-) \\
\hline
 & x - 2 \\
 & x - 2 \\
 & (-) \quad (+) \\
\hline
 & 0
\end{array}$$

$\therefore g(x) = x^2 - x + 1$

UNIT 10: QUADRATIC EQUATIONS

I)

1) B) – 5, 3 2) (B) 2 3) (C) – 8 4) (A) $x^2 + x - 30 = 0$

II)

1) $b^2 - 4ac$

2) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3) $x^2 - 100 = 0$

$x^2 = 100$

$x = \pm\sqrt{100}$

$x = \pm 10$

4) The roots are real and distinct.

III)

1) $x^2 + 5x + 6 = 0$

$x^2 + 3x + 2x + 6 = 0$

$x(x+3) + 2(x+3) = 0$

$(x+3)(x+2) = 0$

$x + 3 = 0 \ and \ x + 2 = 0$

$x = -3 \ and \ x = -2$

2) $x^2 + 4x + 4 = 0$

$a = 1, \quad b = 4, \quad c = 4$

$Discriminant = b^2 - 4ac$

$= 4^2 - 4(1)(4)$

$= 16 - 16$

$= 0$

3) $x^2 - 7x + 6 = 0$

$a = 1, b = -7, c = 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(6)}}{2(1)}$$

$x = \frac{7 \pm \sqrt{49 - 24}}{2}$

$x = \frac{7 \pm \sqrt{25}}{2} = \frac{7 \pm 5}{2}$

$x = \frac{7+5}{2}$ or $x = \frac{7-5}{2}$

$x = \frac{12}{2}$ $x = \frac{2}{2}$

$x = 6$ $x = 1$

4) $x^2 - 3x + 1 = 0$

$a = 1, \ b = -3, \quad c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9 - 4}}{2}$

$x = \frac{3 \pm \sqrt{5}}{2}$

$x = \frac{3+\sqrt{5}}{2}$ or $x = \frac{3-\sqrt{5}}{2}$

5) If the roots of the equation are equal then

$b^2 - 4ac = 0$

$a = 1, \quad b = -k, \quad c = 4$

$b^2 - 4ac = 0$

$(-k)^2 - 4(1)(4) = 0$

$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm\sqrt{16}$

$k = \pm 4$

6) $a = 2, \ b = -5, \ c = -1$

Discriminant = $b^2 - 4ac$

$= (-5)^2 - 4(2)(-1)$

$= 25 + 8$

$= 33$

The roots are real and distinct

IV)

1) Let the present age of sister be 'x' years

Therefore present age of the girl will be '$2x$' years

The product of their ages hence 4 years =$(x + 4)(2x + 4)$

$\therefore (x + 4)(2x + 4) = 160$

$2x^2 + 4x + 8x + 16 = 160$

$2x^2 + 4x + 8x + 16 - 160 = 0$

$2x^2 + 12x - 144 = 0$

$x^2 + 6x - 72 = 0$

$x^2 + 12x - 6x - 72 = 0$

$x(x + 12) - 6(x + 12) = 0$

$x = -12 \ or \ x = 6$

Age cannot be negative => $x = 6$

$\therefore$ Girl's present age is 12 years and

present age of her sister is 6 years

2) Let the base be ‘x‘ cm and altitude be $(x-7)$ cm and hypotenuse 13 cm
By Pythagoras theorem.

$13^2 = (x-7)^2 + x^2$

$169 = x^2 + 49 - 14x + x^2$

$2x^2 - 14x - 120 = 0$

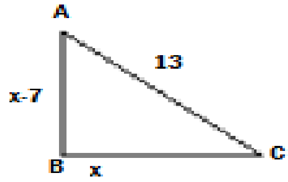
$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x-12) + 5(x-12) = 0$

$x - 12 = 0 \quad or \quad x + 5 = 0$

$x = 12 \quad or \quad x = -5$



Base is 12 cm and Altitude is 5 cm.

V)

1) Let the speed of passenger train be ‘x’ km/h

$\therefore$ The speed of express train is $(x + 11)\ km/h$

Distance travelled = 132km

We know that Speed = $\frac{\text{Distance travelled}}{\text{Time taken}}$ or Time taken = $\frac{\text{Distance travelled}}{\text{Speed}}$

Time taken by passenger train = $\frac{132}{x}$ h

Time taken by express train = $\frac{132}{x+11}\ h$

Difference in time taken of two trains = 1 h

$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$

$132(x + 11) - 132x = x(x + 11)$

$132x + 1452 - 132x = x^2 + 11x$

$x^2 + 11x - 1452 = 0$

$x^2 + 44x - 33x - 1452 = 0$

$x(x + 44) - 33(x + 44) = 0$

$(x + 44)(x - 33) = 0$

$x + 44 = 0\ , \quad x - 33 = 0$

$x = -44\ , \quad x = 33$

The speed cannot be negative.

$\therefore$ The average speed of passenger train is 33 km/h

$\therefore$ The average speed of express train is 44 km/h

2) Let the original duration of the tour be ‘x’ days.

Given, $\frac{4200}{x} - \frac{4200}{x+3}$ =70

$4200\left(\frac{1}{x} - \frac{1}{x+3}\right) = 70$

$\frac{(x+3)-x}{x(x+3)} = \frac{70}{4200}$

$x(x+3) = 180$

$x^2 + 3x - 180 = 0$

$x^2 + 15x - 12x - 180 = 0$

$(x + 15)(x - 12) = 0$

$x + 15 = 0 \;\; or \; x - 12 = 0$

$x = -15 \quad or \;\; x = 12$

number of days can't be negative => $x = 12$

∴ **Original duration of the tour is 12 days.**

3) Let the length of the shorter side of the rectangular field be 'x' m

∴ the length of the longer side of the rectangular field is $(x + 30)\,m$ and

The length of the diagonal of the rectangular field is $(x + 60)\,m$

According Pythagoras theorem

The length of the diagonal =$\sqrt{x^2 + (x+30)^2}$

$$\sqrt{x^2 + (x+30)^2} = x + 60$$

$$x^2 + (x + 30)^2 = (x + 60)^2$$

$$x^2 + x^2 + 2(x)(30) + (30)^2 = x^2 + 2(x)(60) + (60)^2$$

$$2x^2 + 60x + 900 = x^2 + 120x + 3600$$

$$2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$$

$$x^2 - 60x - 2700 = 0$$

$$x^2 - 90x + 30x - 2700 = 0$$

$$x(x - 90) + 30(x - 90) = 0$$

$$(x - 90)(x + 30) = 0$$

$$x - 90 = 0,\; x + 30 = 0$$

$$x = 90, \quad x = -30$$

∴ The length of the shorter side of the rectangular field is $x = 90\,m$

∴The length of the longer side of the rectangular field is $x + 30 = 90 + 30 = 120\,m$

UNIT 11: INTRODUCTION TO TRIGONOMETRY

I. Multiple choice questions

1. A) $sin^2\theta$ 2. D) 60^0 3. B) $\frac{5}{3}$ 4. A) 0 5. C) 1

II. One mark questions

1. $tan\theta \text{ x} \frac{1}{tan\theta} = 1$

2. $sec\ A = \frac{2}{\sqrt{3}}$

$sec60° = \frac{2}{\sqrt{3}}$

$\therefore A = 60^0$

3. $(\text{cosec } 31^0 - \sec 59^0) = \text{cosec } (90^0 - 59^0) - \sec 59^0$

$= \sec 59^0 - \sec 59^0 = 0$

4. $\frac{1-\tan 45^0}{1+tan45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

5. $\boldsymbol{tan\theta} = \frac{sin\theta}{\cos\theta} = \frac{\frac{2}{\sqrt{3}}}{\frac{3}{\sqrt{3}}} = \frac{2}{3}$

III. Two marks questions

1. $\cot(90^0 - 2A) = \cot(A - 18^0)$

$90^0 - 2A = A - 18^0$

$90^0 + 18^0 = A + 2A$

$3A = 108^0$

$A = 36^0$

2. $tan(A-B) = \frac{tanA - \tan B}{1+\tan A.\tan B}$

$LHS = tan(60° - 30°) = tan30°$

$LHS = \frac{1}{\sqrt{3}}$

$RHS = \frac{tan60^0 - \tan 30^0}{1 + \tan 60^0 . \tan 30^0}$

$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\ X\frac{1}{\sqrt{3}}}$

$= \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$

$RHS = \frac{1}{\sqrt{3}}$

$\therefore LHS = RHS$

3. $sin\ (90° - 72°)\ -\ cos72° -\ cos(90° - 72°)\ +\ sin72°$

$= cos72° - cos\ 72°\ -\ sin72°\ +\ sin72°\ =\ 0$

4. LHS = $\frac{sin\theta}{1-\cos\theta} = \frac{sin\theta}{1-\cos\theta}\ X\ \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{sin\theta\ (1+\cos\theta)}{1-\boldsymbol{cos^2}\theta}$

$= \frac{sin\theta\ (1+\cos\theta)}{\boldsymbol{sin^2}\theta}$

$= \frac{1+\cos\theta}{sin\theta}$

$= \frac{1}{sin\theta} + \frac{\cos\theta}{sin\theta}$

$= cosec\ \theta\ +\ cot\ \theta$

IV. Three marks questions

1. $\sqrt{\frac{1+sinA}{1-sinA}} x \sqrt{\frac{1+sinA}{1+sinA}}$

$= \sqrt{\frac{(1+sinA)(1+sinA)}{(1-sinA)(1+sinA)}}$

$= \sqrt{\frac{(1+sinA)^2}{1-sin^2 A}}$

$= \sqrt{\frac{(1+sinA)^2}{cos^2 A}}$

$= \frac{1+sin\mathrm{A}}{\cos\mathrm{A}}$

$= \frac{1}{\cos\mathrm{A}} + \frac{sin\mathrm{A}}{\cos\mathrm{A}}$

$= secA + tanA$

2. $tan(A + B)\ = \sqrt{3}$ $\quad$ $tan(A - B) = \frac{1}{\sqrt{3}}$

$tan\ 60° =\ \sqrt{3}$ $\quad$ $tan\ 30°\ =\ \frac{1}{\sqrt{3}}$

$A\ + B\ = 60°$ ----- (1) $\quad$ $A - B\ = 30°$ -----(2)

By solving equation 1 and 2, we get A= 45^0 and B=15^0

3. L.H.S = $\frac{sin(90-\theta)}{1+\sin\theta} + \frac{cos\theta}{1-cos(90-\theta)}$

$= \frac{cos\theta}{1+\sin\theta} + \frac{cos\theta}{1-sin\theta}$

$= \frac{cos\theta\ (1-\sin\theta)+cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{cos\theta - \cos\theta .\sin\theta + cos\theta + \cos\theta.\sin\theta)}{1^2 - sin^2\theta}$

$= \frac{2cos\theta}{cos^2\theta} = \frac{2}{\cos\theta}$

$=2sec\theta$

4. $\frac{1-cos\theta}{1+\cos\theta}x\frac{1-cos\theta}{1-\cos\theta} = \frac{(1-cos\theta)^2}{1^2-\cos^2\theta}$

$= \frac{1^2+cos^2\theta-2\cos\theta}{sin^2\theta}$

$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - \frac{2cos\theta}{\sin^2\theta}$

$= \text{cosec}^2\theta + \cot^2\theta - \frac{2cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}$

$= \text{cosec}^2\theta + \cot^2\theta - 2\cot\theta.\text{cosec}\theta$

$= (\text{cosec}\,\theta - \cot\theta)^2$

5. $x^2 - y^2 = (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2a\sec\theta.\, b\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2a\tan\theta.\, b\sec\theta$

$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$

$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$= (\sec^2\theta - \tan^2\theta)(a^2 - b^2)$

$= 1(a^2 - b^2) = (a^2 - b^2)$

6. L.H.S $= (\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A.\,\text{cosec}A + \cos^2 A + \sec^2 A + 2\cos A.\sec A$

$= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 2(\sin A.\,\text{cosec}A + \cos A.\sec A)$

$= 1 + \text{cosec}^2 A + \sec^2 A + 2(1+1)$

$= 1 + \text{cosec}^2 A + \sec^2 A + 2(2) = 5 + 1 + \cot^2 A + 1 + \tan^2 A = 7 + \cot^2 A + \tan^2 A$

V Four or five marks questions

1. $\frac{\text{cosec}A+1}{\text{cosec}A-1}x\frac{\text{cosec}A+1}{\text{cosec}A+1}$

$= \frac{(\text{cosec}A+1)^2}{\text{cosec}^2A-1^2}$

$= \frac{\text{cosec}^2A+1^2+2\,\text{cosec}A}{\cot^2A}$

$= \frac{\text{cosec}^2 A}{\cot^2 A} + \frac{1}{\cot^2A} + \frac{2\,\text{cosec}A}{\cot^2A}$

$= \frac{1}{\sin^2A} \text{ x } \tan^2A + \tan^2A + 2\text{x}\,\frac{1}{\sin A}\,\text{x}\tan^2A$

$= \frac{1}{\sin^2A} \text{ x } \frac{\sin^2 A}{\cos^2A} + \tan^2A + \frac{1}{\sin A} \text{ x } \frac{\sin^2 A}{\cos^2 A}$

$= \sec^2 A + \tan^2A + 2\tan A.\sec A$

$= (\sec A + \tan A)^2$

2. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$$\text{L.H.S} = \frac{tan\theta}{1-cot\theta} + \frac{cot\theta}{1-tan\theta}$$

$$= \frac{\frac{sin\theta}{cos\theta}}{1-\frac{cos\theta}{sin\theta}} + \frac{\frac{cos\theta}{sin\theta}}{1-\frac{sin\theta}{cos\theta}}$$

$$= \frac{\frac{sin\theta}{cos\theta}}{\frac{sin\theta - cos\theta}{sin\theta}} + \frac{\frac{cos\theta}{sin\theta}}{\frac{cos\theta - sin\theta}{cos\theta}}$$

$$= \frac{sin^2\theta}{cos\theta(\sin\theta - \cos\theta)} + \frac{cos^2\theta}{sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{sin^2\theta}{cos\theta(\sin\theta - \cos\theta)} - \frac{cos^2\theta}{sin\theta(-\cos\theta + \sin\theta)}$$

$$= \frac{sin^3\theta - cos^3\theta}{\sin\theta\, cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(sin\theta - \cos\theta)(sin^2\theta + cos^2\theta + sin\theta.cos\theta)}{\sin\theta\, cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{sin^2\theta}{sin\theta\,.\cos\theta} + \frac{cos^2\theta}{sin\theta\,.\cos\theta} + \frac{sin\theta\,.\cos\theta}{sin\theta\,.\cos\theta}$$

$$= tan\theta + cot\theta + 1 = \text{RHS}$$

$$3.\text{L.H.S} = \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2$$

$$= \frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+1+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{2(1+\sin\theta-\sin\theta\cos\theta-\cos\theta)}{2(1+\sin\theta+\sin\theta\cos\theta+\cos\theta)}$$

$$= \frac{(1-\cos\theta+\sin\theta-\sin\theta\cos\theta)}{(1+\cos\theta+\sin\theta+\sin\theta\cos\theta)}$$

$$= \frac{(1-\cos\theta)+\sin\theta(1-\cos\theta)}{(1+\cos\theta)+\sin\theta(1+\cos\theta)}$$

$$= \frac{(1-\cos\theta)(1+\sin\theta)}{(1+\cos\theta)(1+\sin\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

UNIT-12 : SOME APPLICATIONS OF TRIGONOMETRY

Two Marks Questions

1. Let A be the point of observation and C be the top of the building.

Then AB=$100\sqrt{3}ft$ and $\underline{|A}$ =30^0

$tanA = \frac{BC}{AB}$

$$tan30^0 = \frac{BC}{100\sqrt{3}}$$

$\frac{1}{\sqrt{3}} = \frac{BC}{100\sqrt{3}}$

= > BC= 100m

∴ **height of the building is 100ft**.

2. Let P be the point on the ground where thread is tied and

R be the position of kite.

Then QR=$50\sqrt{3}m$ and PR= 100m

$sinP = \frac{QR}{PR}$

$$sinP = \frac{50\sqrt{3}}{100}$$

$$sinP = \frac{\sqrt{3}}{2}$$

= > $\underline{|P}$= 60^0

∴**Thread makes an angle of 60^0 with the ground.**

Three or Four Marks Questions.

1 $In\ \Delta OAC, tan30° = \frac{AC}{OA}$

$\frac{1}{\sqrt{3}} = \frac{x}{30}$

$x = \frac{30}{\sqrt{3}} = 10\sqrt{3}\ m$

$Again\ in\ \Delta OAC,\ \ cos30° = \frac{OA}{OC}$

$\frac{\sqrt{3}}{2} = \frac{30}{y}$

$y = \frac{60}{\sqrt{3}} = 20\sqrt{3}\ m$

Height of the tree $= (x + y) = \left(10\sqrt{3} + 20\sqrt{3}\right)$

∴ **Height of the tree** $= \mathbf{30\sqrt{3}\ m}$.

2 $In\ \Delta ACQ,\ \ tan 30° = \frac{QC}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{30-1.5}{AC}$$

$$AC = 28.5\sqrt{3}\ m$$

$In\ \Delta BCQ,\quad \tan 60° = \frac{QC}{BC}$

$$\sqrt{3} = \frac{30-1.5}{BC}$$

$$BC = \frac{28.5}{\sqrt{3}}\ m$$

$AB = AC - BC$

$AB = 28.5\sqrt{3} - \dfrac{28.5}{\sqrt{3}}$

$\therefore \mathbf{AB = 19\sqrt{3}\ m}$

3

Let Height of the building is "***h***" m

$In\ \Delta ABC,\ \ tan 60° = \dfrac{BC}{AB}$

$\sqrt{3} = \dfrac{50}{AB}$

$AB = \frac{50}{\sqrt{3}}$ . . . . (1)

$In\ \Delta ABD,\ \ tan 30° = \dfrac{AD}{AB}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{AB}$

$AB = h\sqrt{3}$ . . . .(2)

From equation 1 and 2

$\frac{50}{\sqrt{3}} = h\sqrt{3}$

$\therefore \mathbf{h = \dfrac{50}{3}\,m}$.

4 In ΔAOQ, $\tan 45° = \frac{OA}{OQ}$

$1 = \dfrac{75}{OQ}$

$OQ = 75$

$In\ \Delta AOP,\ \ tan 30° = \dfrac{OA}{OP}$

$\dfrac{1}{\sqrt{3}} = \dfrac{75}{OP}$

$OP = 75\sqrt{3}$

$PQ = OP - OQ$

$PQ = 75\sqrt{3} - 75$

$\therefore \mathbf{PQ = 75(\sqrt{3} - 1)m}$.

5 Height of the tower be 'h' m

In ΔABC, $tan\theta = \frac{h}{4} ----(i)$

In ΔABD, $tan(90° - \theta) = \frac{h}{9}$

$cot\theta = \frac{h}{9} ----(ii)$

Multiplying (i) and (ii)

$tan\theta x \; cot\theta = \frac{h}{4}$ x $\frac{h}{9}$

$1 = \frac{h^2}{36}$

$h^2 = 36$

$h = 6m$

∴**Height of the tower is 6 meter.**

6 In ΔADE, $tan60° = \frac{h}{AD}$

$\sqrt{3} = \frac{h}{x}$

$h = x\sqrt{3}$ -------(i)

In ΔABC, $tan30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{10}{x}$

=> $x = 10\sqrt{3}$ ----(ii)

Distance of the ship from the hill =10$\sqrt{3}$ m

Substituting (ii) in (i) gives $h = 10\sqrt{3} \; x \; \sqrt{3}$

$h = 30 \; m$

=> **Height of the hill = 30+10 = 40 m.**

7 Let P and Q be the two positions of plane

A be the point of observation

In Δ ABP, $tan60° = \frac{PB}{AB}$ => $\sqrt{3} = \frac{3600\sqrt{3}}{AB}$

$AB = 3600 \; m$

In Δ ACQ, $tan30° = \frac{CQ}{AC}$ => $\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$

AC = 10800 m

BC = 10800 – 3600

BC = 7200 m

but BC=PQ => Distance travelled is 7200 m

Speed of the plane = $\frac{7200}{30}$ = $240 \; m/s$

UNIT 13 : STATISTICS

MULTIPAL CHOICE (1 MARKS)

1) B. 5 2) C. 11 3) D. 2 4) C. median 5) A. 2

VERY SHORT ANSWER QUESTIONS (1 MARKS)

1. Class mark = $\frac{lower\ limit + upper\ limit}{2} = \frac{40+50}{2} = \frac{90}{2} = 45$
2. 3 Median = Mode + 2 Mean

THREE MARKS QUESATIONS

1.

Class interval	*Frequency*	x_i	$f_i x_i$
0-10	2	5	10
10-20	6	15	90
20-30	5	25	125
30-40	3	35	105
40-50	4	45	180
	$\sum f_i = 20$		$\sum f_i x_i = 510$

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{510}{20} = 25.5$

2.

Class interval	*Frequency*	x_i	$d_i = x_i - a$	$f_i d_i$
10-20	7	15	-20	-140
20-30	10	25	-10	-100
30-40	6	35	0	0
40-50	8	45	+10	+80
50-60	9	55	+20	+110
	$\sum f_i = 40$			$\sum f_i d_i = 1040$

Mean = a + $\frac{\sum f_i d_i}{\sum f_i} = 35 + \frac{1040}{40} = 35 + 26 = 61$

3

CI	f	x_i	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
200-250	7	225	-150	-3	-21
250-300	3	275	-100	-2	-6 = -37
300-350	10	325	-50	-1	-10
350-400	6	375	0	0	0 = - 9
400-450	5	425	+150	+3	+15
450-500	4	475	+100	+2	+8 = +28
500-450	5	525	+50	+1	+5
	$\sum f_i = 40$				$\sum f_i u_i = -9$

Assumed mean = a = 375   h = 50

Mean = a + $\left[\frac{\sum f_i u_i}{\sum f_i}\right] \times h$

= 375 + $\left[\frac{-9}{40}\right] \times 50$

= 375 - $\frac{45}{4}$

= 375 – 11.25

Mean= 363.75

4)

Class interval	*Frequency*	Cumulative frequency
0-10	4	4
10-20	7	4 + 7 = 11
20-30	13	11 + 13 = 24
30-40	9	24 + 9 = 33
40-50	3	33 + 3 = 36
	N = 36	

$\frac{n}{2}$ = 18, l = 20, f = 13, cf = 11, h = 10

Median$= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 20 + \left[\frac{18-11}{13}\right] \times 10$

= 20 + $\frac{70}{13}$

= 20 + 5.38

= 25.38

5)

Class interval	*Frequency*	Cumulative frequency
10-15	8	8
15-20	4	8 + 4 = 12
20-25	6	12 + 6 = 18
25-30	2	18 + 2 = 20
30-35	6	20 + 6 = 26
	N = 26	

$\frac{n}{2}$ = 13, l = 20, f = 6, cf = 12, h = 5

Median$= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 20 + \left[\frac{13-12}{6}\right] \times 5$

= 20 + $\frac{5}{6}$

= 20 + 0.83

= 20.83

f_1 = 11, f_2 = 6, f_0 = 9,   l = 60,   h = 10

6)

Class interval	*Frequency*
30-40	4
40-50	7
50-60	9
60-70	11
70-80	6
80-90	2

f_1 = 11, f_2 = 6, f_0 = 9, l = 60, h = 10

$$\textbf{\textit{Mode}} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{11-9}{2(11)-9-6}\right) \times 10$$

$$= 60 + \left(\frac{2}{22-15}\right) \times 10$$

$$= 60 + \frac{20}{7} = 60 + 2.85$$

$$= 62.85$$

7)

Class interval	*Frequency*
10-20	8
20-30	12
30-40	5
40-50	17
50-60	3
60-70	6

f_1 = 17, f_2 = 3, f_0 = 5, l = 40, h = 10

$$\textbf{\textit{Mode}} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{17-5}{2(17)-5-3}\right) \times 10$$

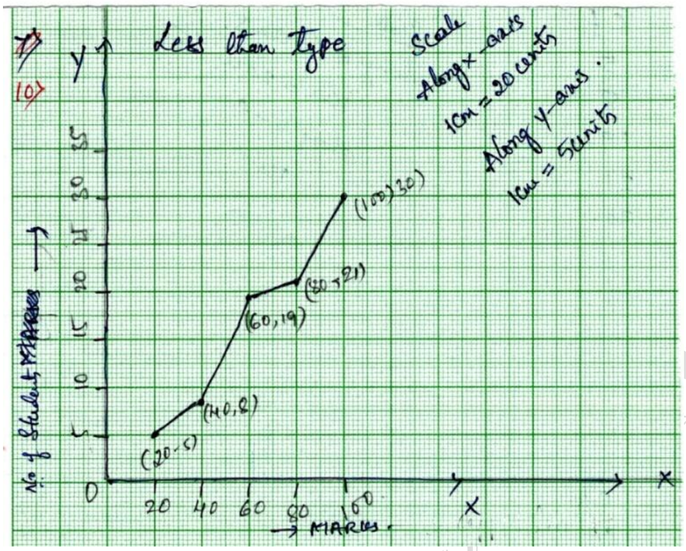
$$= 40 + \left(\frac{12}{34-8}\right) \times 10$$

$$= 40 + \frac{120}{26} = 40 + 4.62$$

$$= 44.62$$

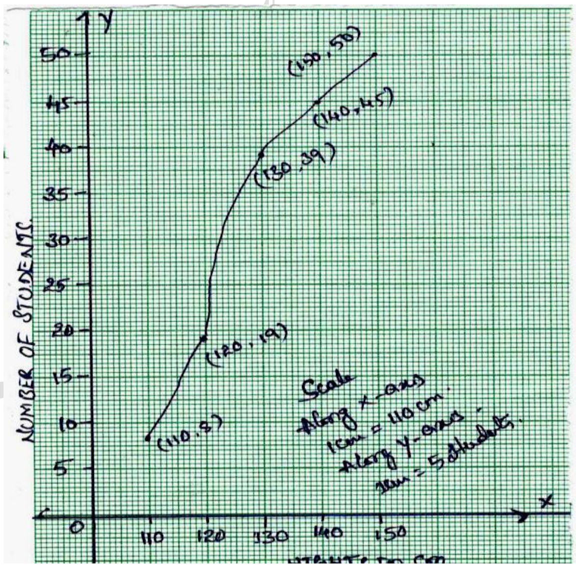
8.

Marks	*Number of students*
Less than 20	5
Less than 40	8
Less than 60	19
Less than 80	21
Less than 100	30



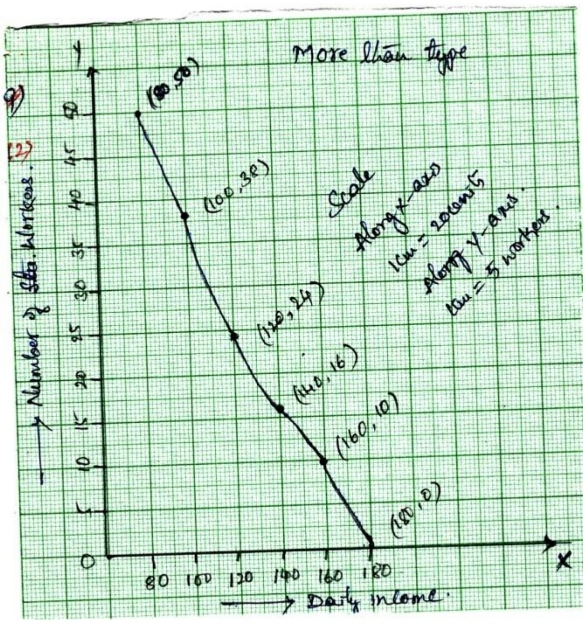
9.

Heights in cm	Number of students
Less than 110	8
Less than 120	19
Less than 130	39
Less than 140	45
Less than 150	50



10.

Daily income	*Number of workers*
More than 80	50
More than 100	38
More than 120	24
More than 140	16
More than 160	10
More than 180	0



UNIT-14: PROBABILITY

I)

1) (C) impossible event 2) (A) 0.3 3) (C) $0 \leq P(A) \leq 1$

II)

1) One 2) One 3) $\frac{1}{2}$

III)

1) Number of all possible outcomes,

$n(S) = 4 + 5 + 8 = 17$

Let A be the event of taking out the white marble.

$\therefore n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\therefore P(A) = \frac{5}{17}$$

2) Number of all possible outcomes, $n(S) = 6$

Let A be the event of getting an even number.

$\therefore n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{3}{6} \quad \therefore P(A) = \frac{1}{2}$$

3) Number of all possible outcomes,

$n(S) = 12 + 132 = 144$

Let A be the event of taking out a defective pen.

$\therefore n(A) = 12$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{12}{144} \quad \therefore P(A) = \frac{1}{12}$$

4) Number of all possible outcomes,

$n(S) = 90$

Let A be the event of taking out a disc bearing a perfect square number.

$\therefore n(A) = 9$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{9}{90}$$

$$\therefore P(A) = \frac{1}{10}$$

III)

1) Number of all possible outcomes,

$n(S) = 36$

Let A be the event of getting the numbers whose sum is less than 7.

$\therefore A = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)$

$(3,3), (4,1), (4,2), (5,1)\}$

$=> n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{15}{36} \quad \therefore P(A) = \frac{5}{12}$$

2) Let there be x blue balls in the bag.

$\therefore$ Total number of balls in the bag $= 5 + x$

Probability of drawing Red ball; $P(R) = \frac{5}{5+x}$

Probability of drawing Blue ball; $P(B) = \frac{x}{5+x}$

Given, $P(B) = 2P(R)$

$$\frac{x}{5+x} = 2\left(\frac{5}{5+x}\right)$$

$\frac{x}{5+x} = \frac{10}{5+x}$ => $\quad x = 10$

$\therefore$ There are 10 blue balls in the bag.

UNIT 15: SURFACE AREAS AND VOLUMES

I)

1) (B) $4\pi r^2$

2) (B) 308 cm^3

3) (C) remain unaltered

4) (C) 616 cm^2

5) (D) 192 cm^3

II)

1) $Volume\ of\ Frustum\ of\ cone = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

2) $\frac{TSA(Sphere)}{TSA(Hemisphere)} = \frac{4\pi r^2}{3\pi r^2} = 4 : 3$

3) $TSA = \pi r(r + l)$

4) $Volume\ of\ sphere = \frac{4}{3}\pi r^3$

III)

1) Given: $l = 8 + 8 = 16cm, b = 8cm,\ h = 8cm,\ T.S.A\ Of\ cuboid = ?$

$$T.S.A.\ of\ a\ cuboid = 2[lb + bh + hl]$$
$$= 2[(16)(8) + (8)(8) + (8)(16)]$$
$$\therefore \boldsymbol{T.S.A.\ of\ a\ cuboid = 640\text{cm}^2}$$

2) Given; edge of the cube=9cm => Height of the cone; $h = 9cm$ and

Radius of the cone; $r = \frac{9}{2}cm$

$$Volume\ of\ the\ cone = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\ x\ \frac{22}{7}\ x\ \frac{9}{2}\ x\ \frac{9}{2}\ x\ 9$$
$$= \frac{2763}{14}$$
$$= 190.93\ cm^2$$

IV)

1) $radius\ of\ the\ sphere\ R = 3cm,$

$radius\ of\ the\ wire\ (cylinder) r = 0.1cm$

$length\ of\ the\ wire\ (cylinder)\ h = ?$

$Volume\ of\ cylinder = Volume\ of\ sphere$

$\pi r^2 h = \frac{4}{3}\pi R^3$

$\pi(0.1)^2 h = \frac{4}{3}\pi(3)^3$

$\boldsymbol{0.01\pi h = 36\pi}$

2) $radius\ of\ big\ solid\ sphere\ R = 24\ cm$

$radius\ of\ small\ solid\ sphere\ r = 3cm$

$Number\ of\ small\ solid\ spheres = ?$

$$Number\ of\ small\ spheres = \frac{V(\ big\ sphere)}{V(a\ small\ \ sphere}$$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{R^3}{r^3}$$

$$= \frac{24^3}{3^3}$$

$\therefore$ **The number of small solid sphere = 512**

V)

1) $hemisphere: r = 5cm$

$Cylinder: r_1 = 5cm, h_1 = 20cm$

$Cone: r_2 = 5cm, h_2 = 12cm$

$$Slant\ height: l_2 = \sqrt{{r_2}^2 + {h_2}^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$\therefore\ l_2 = 13cm$$

$TSA\ of\ the\ toy = CSA\ of\ hemisphere + CSA\ of\ cylinder + CSA\ of\ cone$

$$= 2\pi r^2 + 2\pi r_1 h_1\ + \pi r_2 l_2$$

$$= \left(2 \times \frac{22}{7} \times 5^2\right) + \left(2 \times \frac{22}{7} \times 5\ \times 20\right)\ + \left(\frac{22}{7} \times 5 \times 13\right)$$

$$= \frac{22}{7} \times 5\ (10 + 40 + 13)$$

$$= \frac{110}{7} \times 63$$

$\therefore$ ***TSA of the toy*** $= \mathbf{990cm^2}$

2) $Cylinder: H = 2.1m, D = 4m, R = 2m$

$Cone: l = 2.8m, r = 2m$

$TSA\ of\ the\ canvas = CSA\ of\ cylinder + CSA\ of\ cone$

$$= 2\pi RH + \pi rl$$

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 + \frac{22}{7} \times 2 \times 2.8$$

$\therefore$ ***TSA of the canvas*** $= \mathbf{44m^2}$

$Total\ cost\ of\ the\ canvas\ at\ the\ rate\ of\ Rs. 500\ per\ m^2 = Rs. (500 \times 44)$

$\therefore$ ***Total cost of the canvas*** $= \mathbf{Rs. 22000}$

3) $Cylindrical\ container: R = 6cm, H = 15cm$

$Cone: r = 3cm, h = 12cm$

$V_1 = Volume\ of\ container = \pi R^2 H$

$V_1 = \pi \times 6^2 \times 15 = 540\pi\ cm^3$

$V_2 = Volume\ of\ cone + Volume\ of\ hemisphere = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \pi \times 3^2 (12 + 2 \times 3)$$

$$V_2 = 54\pi cm^3$$

$Number\ of\ ice\ cream\ cones = \frac{V_1}{V_2} = \frac{540\pi}{54\pi}$

$\therefore$ $\boldsymbol{Number\ of\ ice\ cream\ cones\ = 10}$

4) $Original\ cone: r_1 = 12cm, h_1 = 20cm$

$Removed\ cone: r_2 = 3cm, h_2 = ?$

$\frac{h_2}{h_1} = \frac{r_2}{r_1}$ $\qquad$ $\frac{h_2}{20} = \frac{3}{12}$

$h_1 = 5cm$

$h = 20 - 5 = 15cm$

$V = \frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 15(12^2 + 3^2 + 12 \times 3)$

$\therefore$ **Volume of the frustum** $= \mathbf{2970cm^3}$