

SIGNALS AND SYSTEMS

Module-1

INTRODUCTION

Signal: A signal is defined mathematically as a function of one or more independent variables, which conveys information on the nature of a physical phenomenon.

$$\text{i.e. } F \{ \underbrace{x_1, x_2, x_3, x_4, \dots, x_n}_{\substack{\downarrow \\ \text{independent variables}}} \}$$

↓
Signal

Anything which is variable is called a signal. ex: AC
Anything which is not variable i.e. constant is not a signal ex: D.C.

→ When the function depends on the single variable then the signal is said to be one dimensional or single variable signal.

ex: Speech signal. [Amplitude varies with time]

→ When the function depends on two or more variables then the signal is said to be multidimensional or multivariable signal.

ex: A photograph. [The intensity and brightness at each point is a function of two independent variables x & y
i.e. Intensity is denoted as $I(x, y)$]

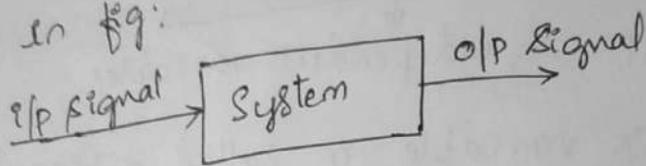
System: A system is defined as an entity, that manipulates one or more signals to accomplish a function, there by yielding new signal.

* The system can also be defined as meaningful interconnection of physical devices, components and operations.

ex: Filter

Filter that is used to reduce the noise in the information bearing signal is also called as System. The operation performed by the sm on the signal is called "processing", which involves elimination of the noise and interferences from the signal.

The interaction b/w signal and the system is as shown in fig:



The i/p signal is also called as excitation and the respective output is the response. The Response is always more desirable than the excitation.

Classification of Signals.

One dimensional signal is said to be single valued signal or function of time. "Single valued" means that for every instant of time there is a unique value of the function. This value may be real number in which that signal is a "Real valued signal" or it may be complex number in which the signal is a "Complex valued signal". The single valued functions can be classified into five categories based on their features. They are

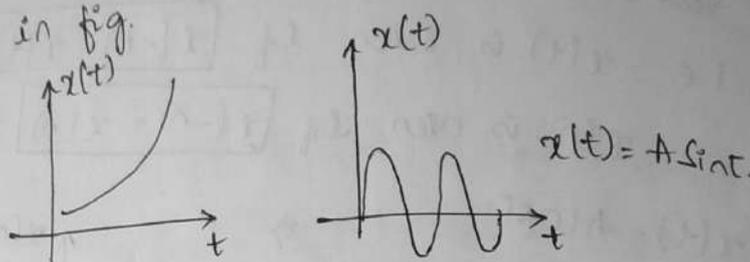
- 1) continuous and Discrete time signals
- 2) Even and odd signals
- 3) Periodic and Non-periodic signals.
- 4) Energy and power signals
- 5) Deterministic and Random signals.

1. Continuous And Discrete time signals.

The signal $x(t)$ is said to be continuous, if it is defined for all the values of 't'. Here 't' is a independent variable and is continuous. And is also called as Analog signal.

ex: 1) Speed signal as a function of time
2) Atmospheric pressure as a function of altitude

The Continuous time (CT) signal is represented as shown in fig.



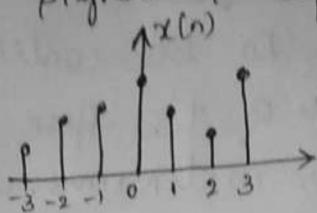
The signal $x(n)$ is said to be Discrete time signal if it is defined only at the discrete instants of time. In this case the independent variable time 't' has discrete values only which are usually uniformly spaced. Discrete time (DT) signal is also called as Digital signal.

A DT signal is often derived from CT signal by the process called Sampling. It is a process of converting CT signal into a DT signal by sampling at uniform rate by the sampling period is T_s and 'n' is an integer that may be +ve or -ve values. Then sampling a CT signal $x(t)$ at time $t = nT_s$ yields a sample value $x(nT_s)$ denoted by $x(n)$. $T_s = 1$ for uniformly sampled signal.

i.e. $x(nT_s) = x(n)$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$

ex: 1) Number of accidents occur in a year
2) Total population \Rightarrow Family size.

The DT signal is represented as shown in fig.



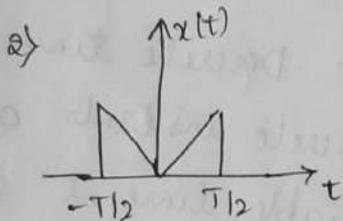
2. Even and Odd Signals.

A signal $x(t)$ or $x(n)$ is said to be even signal if it is identical to its time reversal counterpart or with its reflection about the origin. Or symmetrical about vertical axis.

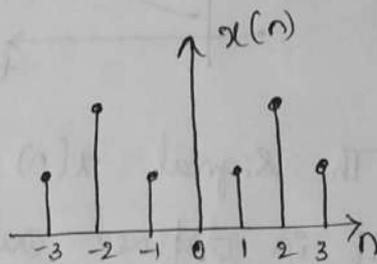
i.e. $x(t)$ is even if $x(-t) = x(t) \quad \forall t$

$x(n)$ is even if $x(-n) = x(n) \quad \forall n$

ex: if $x(t) = A \cos t$



3)



A signal $x(t)$ or $x(n)$ is said to be odd signal if it is antisymmetrical about the origin.

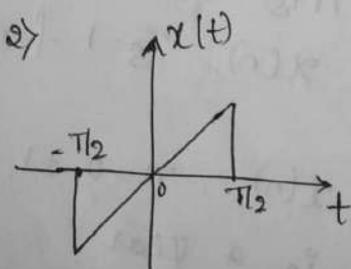
i.e. C.T signal $x(t)$ is said to be odd if

$$x(-t) = -x(t) \quad \forall t$$

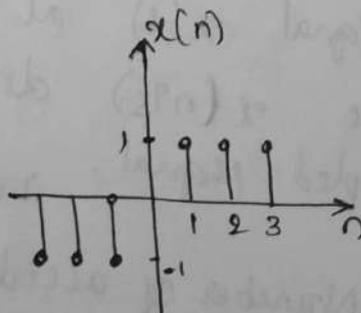
and D.T signal $x(n)$ is said to be odd if

$$x(-n) = -x(n) \quad \forall n$$

ex: if $x(t) = A \sin t$



3)



Decomposition of a signal

any arbitrary signal $x(t)$ can be decomposed into an even & odd signal by applying the corresponding definitions.

Let $x(t)$ be expressed as a sum of two components $x_e(t)$ and $x_o(t)$

$$\text{i.e. } x(t) = x_e(t) + x_o(t) \rightarrow \textcircled{1}$$

$x_e(t)$ = even component

$x_o(t)$ = odd component

put $t = -t$ in eq - ①

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \rightarrow \textcircled{2}$$

Solving for $x_e(t)$ & $x_o(t)$ from ① & ②

we get

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

If the signal is complex-valued signal then a complex valued signal $x(t)$ is said to be conjugate symmetric if $x(-t) = x^*(t) \rightarrow *$

$$\text{let } x(t) = a(t) + j b(t)$$

$a(t)$ is the real part of $x(t)$ and

$b(t)$ is the imaginary part of $x(t)$

$j = \sqrt{-1}$ then complex conjugate of $x(t)$ is

$$x^*(t) = a(t) - j b(t)$$

Substituting for $x(t)$ and $x^*(t)$ in (2) yields

$$a(t) + jb(t) = a(t) - jb(t)$$

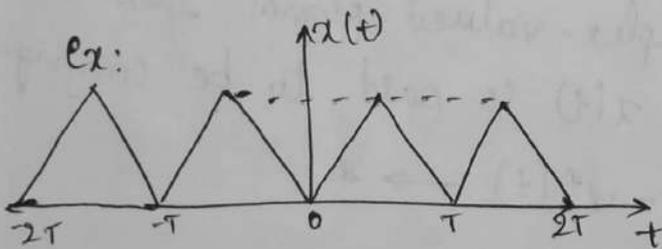
A complex valued signal $x(t)$ is said to be conjugate symmetric if its real part is even and its imaginary part is odd.

3. Periodic And Non-Periodic Signals.

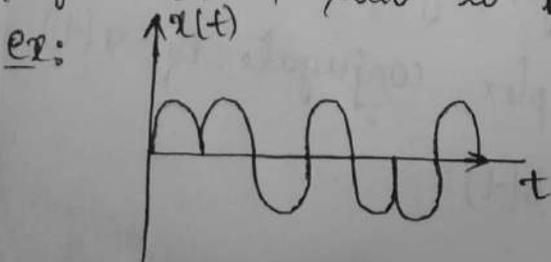
A continuous time signal $x(t)$ is said to be periodic with period T , if there is a positive non zero value of T for which $x(t+T) = x(t)$ for all T ($-\infty$ to ∞)

The smallest value of T for which, the above condition satisfies is called "fundamental period" of $x(t)$. This fundamental period of $x(t)$ is the time taken by the signal to complete its one cycle. The reciprocal of fundamental period ' T ' is known as "Fundamental Frequency" of the signal. i.e. $f = \frac{1}{T}$ Hz

The fundamental angular frequency measured in radians per second is given by $\omega = \frac{2\pi}{T}$ radians

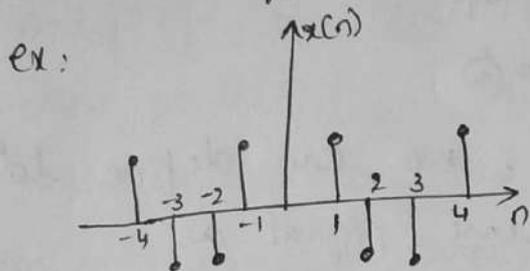


Any signal $x(t)$ for which no value of T satisfies the condition $x(t+T) \neq x(t)$ then the signal $x(t)$ is said to be aperiodic or non-periodic signal.



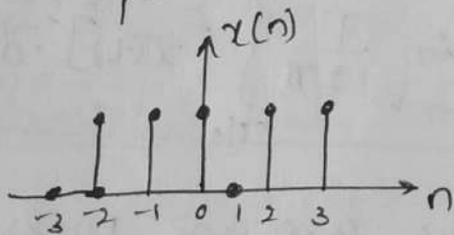
A discrete time signal $x(n)$ is said to be periodic if it satisfies the $x(n) = x(n+N)$ for all n , where N is a +ve integer.

The smallest value of N for which the above condition satisfies is called the fundamental period of the $x(n)$. The angular frequency Ω is $\Omega = \frac{2\pi}{N}$ radians.



period $\Rightarrow N = 4$ samples.

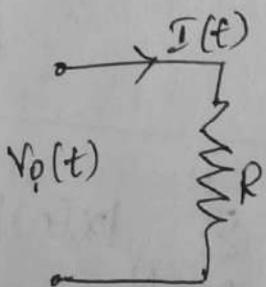
Any signal $x(n)$ for which there is no value of N to satisfy the condition $x[n] = x[n+N]$ then the signal $x(n)$ is called aperiodic or non-periodic.



$$x(n) = \begin{cases} 1 & n = 0, 2, 3, -1, -3 \\ 0 & \text{otherwise} \end{cases}$$

4. Energy Signals and Power Signals.

In electrical sys, signal may represent voltage or current.



Consider voltage $v(t)$ developed across the resistor R , producing current $I(t)$. The instantaneous power dissipated in resistor R is defined by

$$p(t) = \frac{v^2(t)}{R} \rightarrow \textcircled{1}$$

$$\text{equivalently } p(t) = R \cdot I^2(t) \rightarrow \textcircled{2}$$

In both the cases $p(t)$ is proportional to square of amplitude of the signal. further, for a resistor $R=1\Omega$ equations $\textcircled{1}$ & $\textcircled{2}$ takes the same mathematical form. Hence, the instantaneous power signal can be expressed as

$$p(t) = x^2(t) \rightarrow \textcircled{3}$$

On the basis of this, we can define total energy of continuous time signal as,

$$E = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} x^2(t) \cdot dt$$

$$E = \int_{-\infty}^{\infty} x^2(t) \cdot dt, \text{ and}$$

$$\text{average power as } p = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T/2}^{T/2} [x(t)]^2 \cdot dt$$

The square root of the average power p is called the root mean square (RMS) value of the periodic signal $x(t)$.

In case of discrete time signal $x(n)$, the total energy

$$E = \sum_{n=-\infty}^{\infty} [x(n)]^2$$

and average power is

$$p = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

If $x(n)$ is periodic with fundamental period N ,
 average power is

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \quad \text{|||ly for C.T.S}$$

$$P = \frac{1}{T} \int_0^T |x(t)|^2 dt$$

Note:

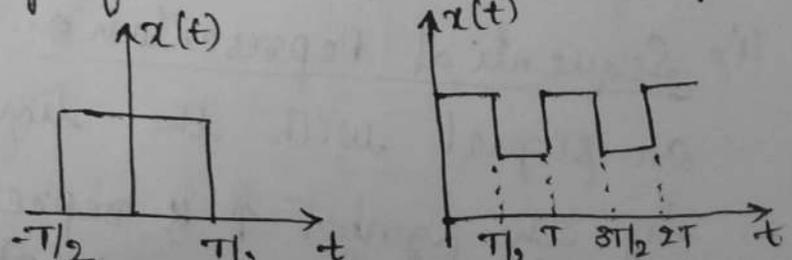
- * A signal is referred to as energy signal if and only if the total energy of the signal satisfies the condition $0 < E < \infty$
- * A signal is referred to as power signal if and only if the average power of the signal satisfies the condition $0 < P < \infty$
- * The energy and power classification of signals are mutually exclusive. In particular, an energy signal has zero time average power, whereas a power signal has infinite energy.

5. Deterministic and Random Signals.

A deterministic signal is a signal about which there is no uncertainty with its value at any time.

These signals are described uniquely by a mathematical expression, graph, & well defined rule, which means that if we know the value of signal at $t = t_1$, we can precisely find its value at any time and other instants

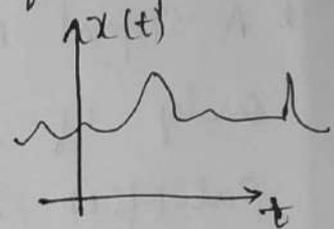
ex:



A Random signal is a signal about which there is a uncertainty about its value before its actual occurrence.

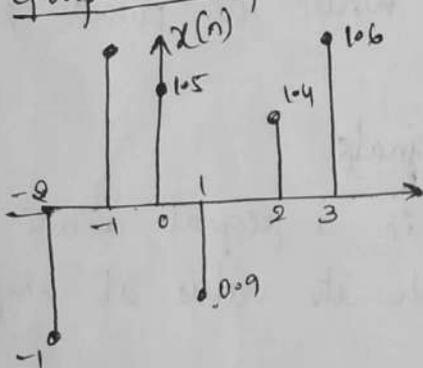
These signals cannot be described by formula or graph yet we know that they exist because they do not appear to possess any definite pattern. These signals are described by their statistical properties like range, average value, probability distribution etc.

ex: The noise generated in an amplifier or a radio receiver or T.V receiver
EEG, ECG, Thermal noise etc



Signal Representation

1) Graphical representation



2) Functional representation

$$x(n) = \begin{cases} 1 & \text{for } n = 0, 4, 6, 8 \\ 1.5 & \text{for } n = 1 \\ 2 & \text{for } n = -2, 2, 8 \\ 1.6 & \text{for } n = -1, 3, 7 \end{cases}$$

3) Tabular representation

n	...	-2	-1	0	1	2	3	4	5	6	7	...
x(n)	...	2	1.6	1.5	2	1.6	1.5	2	4			

4) Sequential Representation. An infinite duration sequence on signal with the time origin ($n=0$) is indicated by the symbol \uparrow & represented as

$$x(n) = \{ \dots, -1, 2, \uparrow 1.5, 0.9, 0.5, \dots \}$$

Basic Operations on Signals [Transformation of Signals]

1) Amplitude Scaling: Let $x(t)$ be a continuous time signal. Then signal $y(t)$ resulting from amplitude scaling is given by

$$y(t) = C \cdot x(t)$$

$C \rightarrow$ is a scaling factor

A physical device that performs amplitude scaling is an amplifier. If $v(t)$ is the voltage applied across resistance R then the current $i(t)$ is given by

$$i(t) = \frac{v(t)}{R} = \frac{1}{R} v(t)$$

$\frac{1}{R}$ is the scaling factor

In case D.T.S., $y(n) = C \cdot x(n)$

2) Addition: If $x_1(t)$ and $x_2(t)$ are the two C.T.S., then

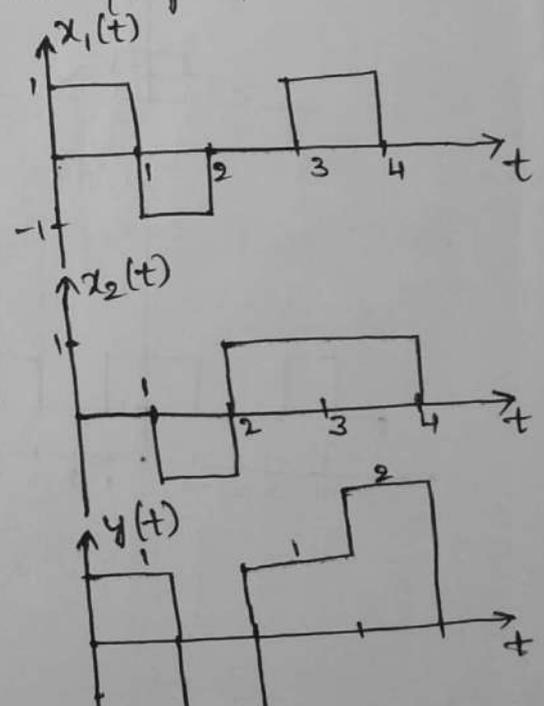
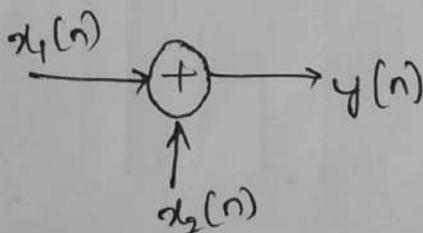
$$y(t) = x_1(t) + x_2(t)$$

A physical device that performs addition is audio mixer, in which the voice signals are combined with music.

In case of D.T.S.

$$y[n] = x_1[n] + x_2[n]$$

Addition is represented as

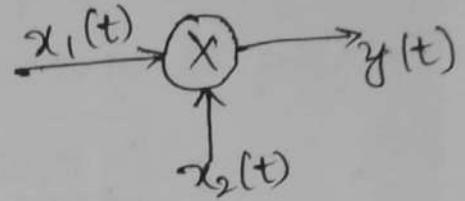


3) Multiplication: If $x_1(t)$ and $x_2(t)$ are the two continuous time signals then $y(t)$ resulting from the multiplication of the signals is

$$y(t) = x_1(t) \cdot x_2(t)$$

Similarly for D.T.S

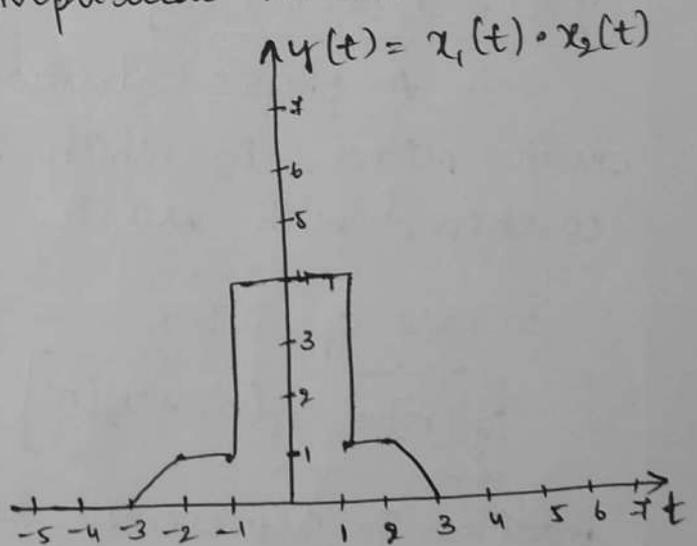
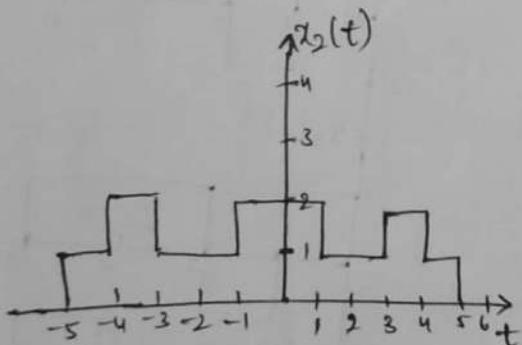
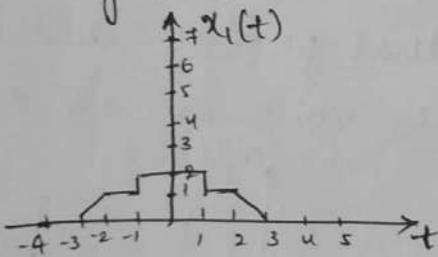
$$y(n) = x_1(n) \cdot x_2(n)$$



The value of $y(t)$ at a given time t is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$

A physical example of a device that performs multiplication is modulator. In which the audio frequency signal is multiplied by a high frequency sinusoidal called carrier signal. The resulting signal $y(t)$ is called Amplitude modulation.

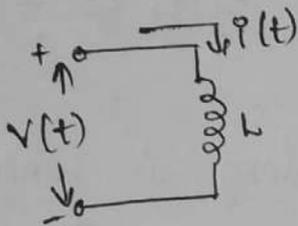
ex:



4) Differentiation: Let $x(t)$ denotes a continuous time signal. The derivative of $x(t)$ w.r.t time is defined by

$$y(t) = \frac{dx(t)}{dt}$$

A physical device that performs differentiation is an Inductor

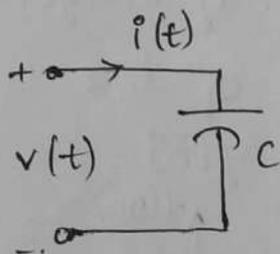


If $i(t)$ the current induced in an inductance L . Then the voltage across it is $v(t)$ is given by

$$v(t) = L \frac{di(t)}{dt} \quad \xrightarrow{x(t)} \left[\frac{d}{dt} \right] \rightarrow y(t)$$

5) Integration: Let $x(t)$ denote a continuous time signal. then the integral of $x(t)$ w.r.t time is denoted by $y(t) = \frac{1}{C} \int_{-\infty}^t x(t) \cdot dt$.

A physical device that performs integration is a capacitor



If $i(t)$ is the current flows through a capacitor C . Then the voltage across it is $v(t)$ is defined as

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(t) \cdot dt$$

Operations performed on the Independent Variable

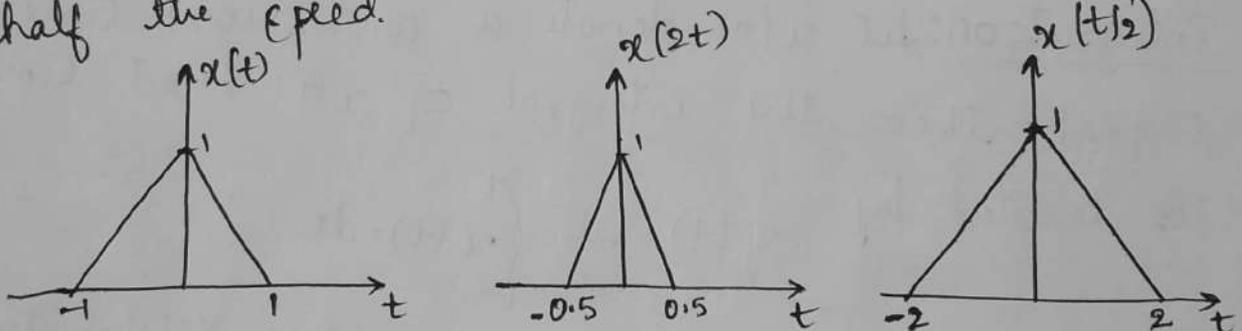
1. Time Scaling: If $x(t)$ is a continuous time signal. Then signal $y(t)$ is obtained by scaling the independent variable time 't' by a factor 'a' and is defined as

$$y(t) = x(at)$$

* If $a > 1$, then the resultant signal $y(t)$ is a compressed version of $x(t)$

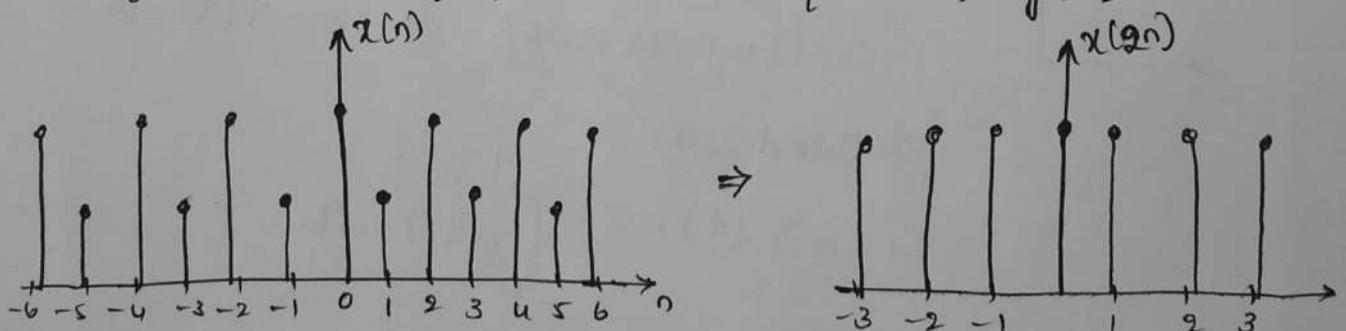
* If $a < 1$, then the resultant signal $y(t)$ is an expanded version of $x(t)$.

ex: a physical example is Tape recording, in which $x(2t)$ is that recording played at twice the speed and $x(t/2)$ is the recording played at half the speed.



$y[n] = x[kn]$ in case of discrete time signal

where $k > 0$ and k can take only integer value. If $k > 1$, then some values of the discrete time signals is lost in $y[n]$



2. Time shifting:

Let $x(t)$ denotes a continuous time signal. Then the time shifted version of $x(t)$ is defined as

$$y(t) = x(t - t_0)$$

where t_0 - time shift.

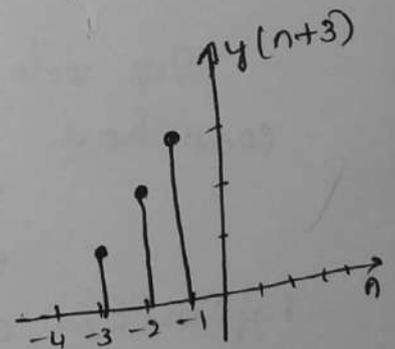
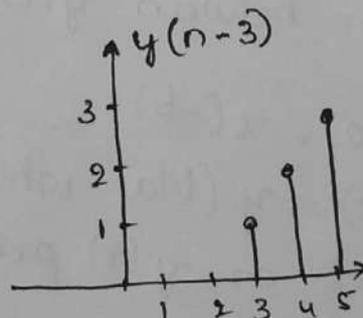
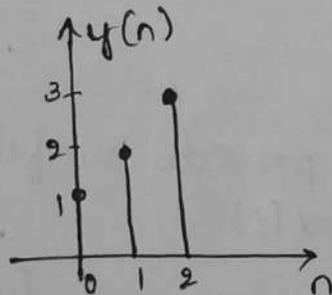
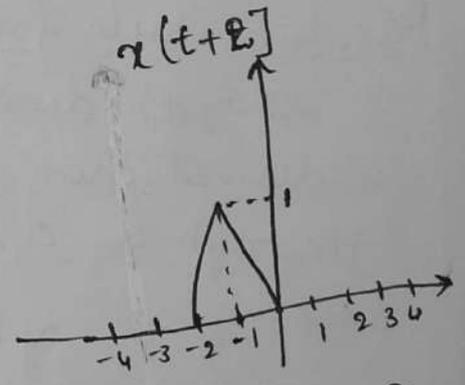
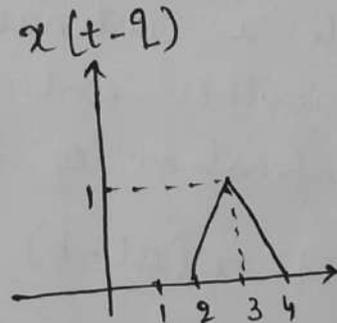
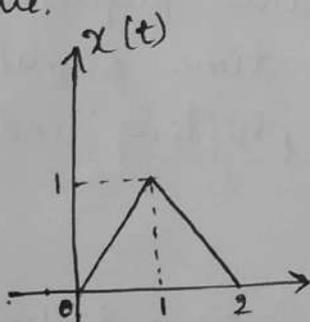
- * If $t_0 > 0$ (+ve), then $x(t - t_0)$ represents the delayed version of $x(t)$. i.e. it is shifted to the right relative to the time axis.
- * If $t_0 < 0$ (-ve), then $x[t - (-t_0)] = x(t + t_0)$ represents the advanced version of $x(t)$. i.e. it is shifted to the left relative to the time axis.

In case of D.T.S

$$y[n] = x[n - n_0]$$

where n_0 is the shift either +ve or -ve integer value.

ex:



3. Reflection & Time Reversal

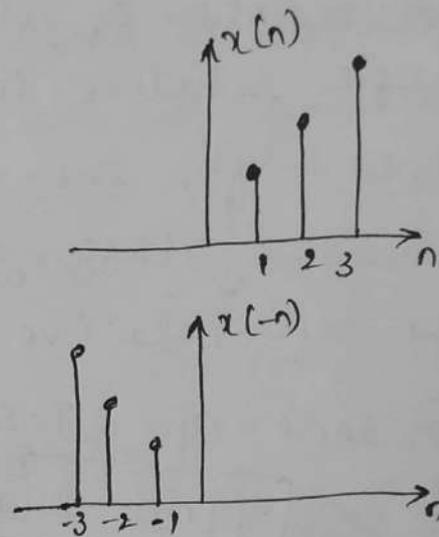
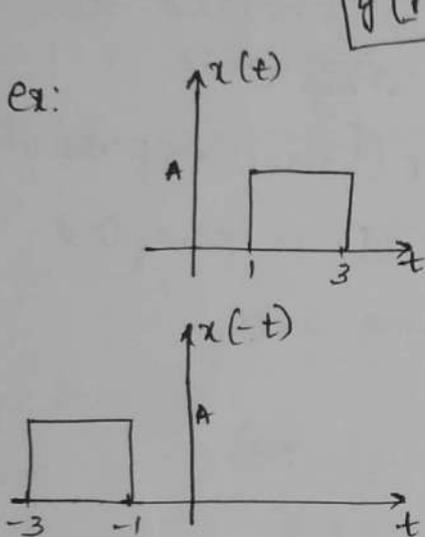
If $x(t)$ is continuous time signal, then $y(t)$ is the signal obtained by replacing time t by $-t$.

$$y(t) = x(-t)$$

In case of D.T.S

$$y[n] = x[-n]$$

ex:



Precedence rule for Time shifting and Time Scaling

Let $y(t)$ denote a continuous time signal that is derived from another continuous time signal $x(t)$ through a combination of time shifting and time scaling

$$y(t) = x(at-b)$$

This relation between $y(t)$ and $x(t)$ satisfies the conditions.

$$y(0) = x(-b)$$

$$x(0) = y(b/a)$$

which provides useful information checks on $y(t)$ and $x(t)$.

To obtain $y(t)$ from $x(t)$ time shift and time scaling operations must be performed in the correct order.

The proper order is based on the fact that the scaling operation always replaces t by at , while time shifting operation always replaces t by $t-b$. Hence the time shifting is performed first on $x(t)$ resulting in

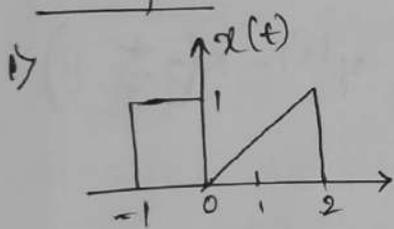
$$v(t) = x(t-b)$$

The time shift has replaced t in $x(t)$ by $t-b$. The next operation is performed on $v(t)$, replacing ' t ' by ' at ' resulting in the desired output.

$$y(t) = v(at)$$

$$y(t) = v(at-b)$$

Examples:

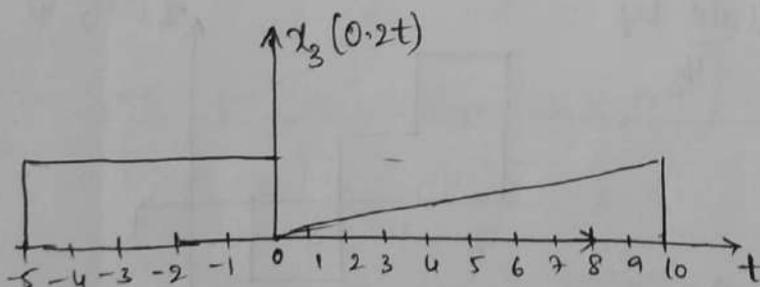
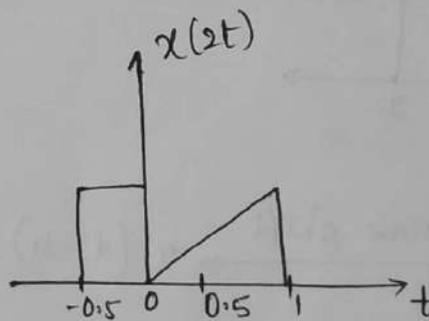
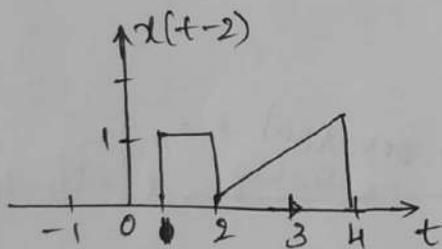


find ① $x_1(t-2)$

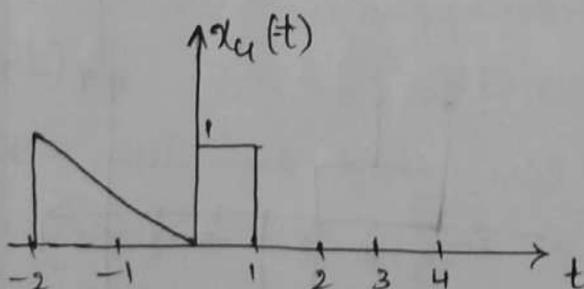
③ $x_3(0.2t)$

② $x_2(2t)$

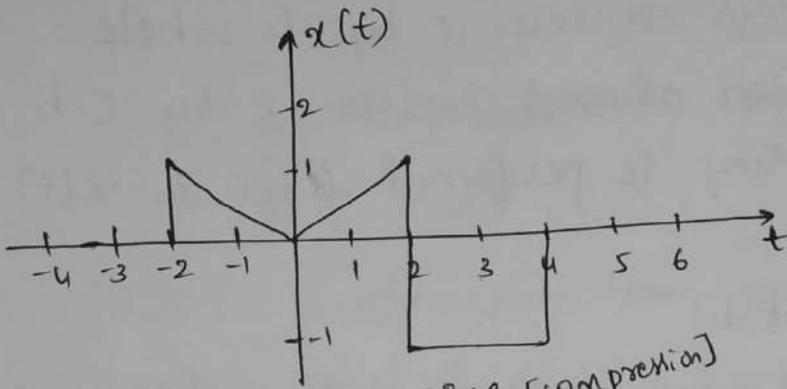
④ $x_4(-t)$



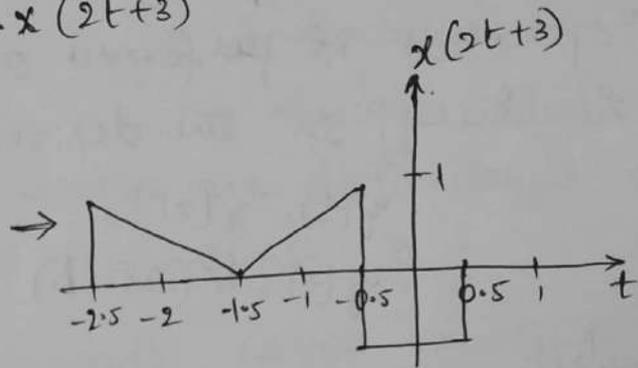
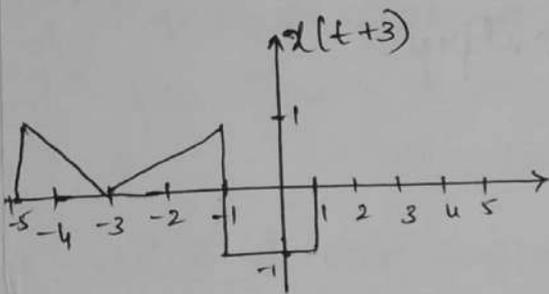
$$0.2t = \frac{1}{5}t$$



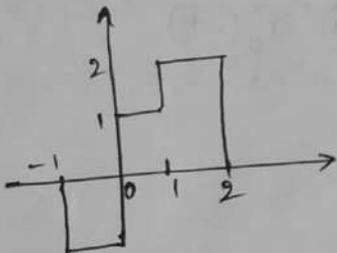
2) Given $x(t)$. find $y(t) = x(2t+3)$



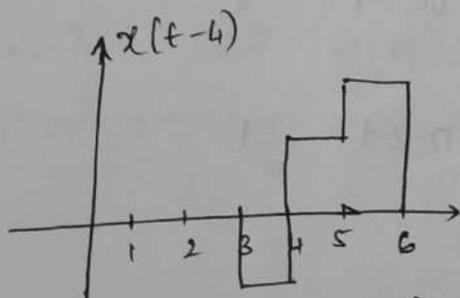
Time Shift $\rightarrow x(t+3)$ Time Scale [compression] $\rightarrow x(2t+3)$



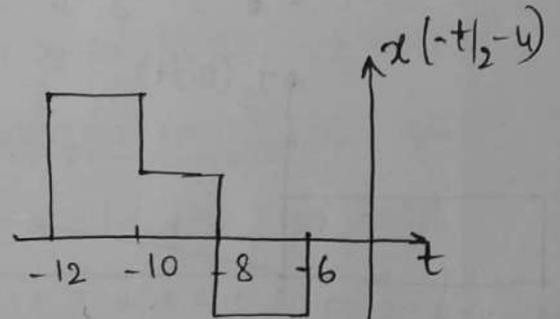
3) Given $x(t)$. find & sketch the signal $y(t) = x(-\frac{t}{2}-4)$



Time shift $\rightarrow x(t-4)$ Time reversal & scaling $\rightarrow x(-\frac{t}{2}-4)$

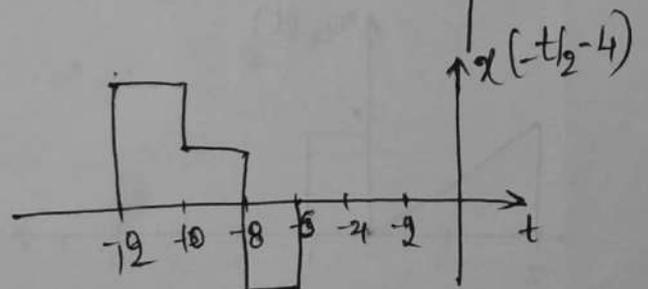
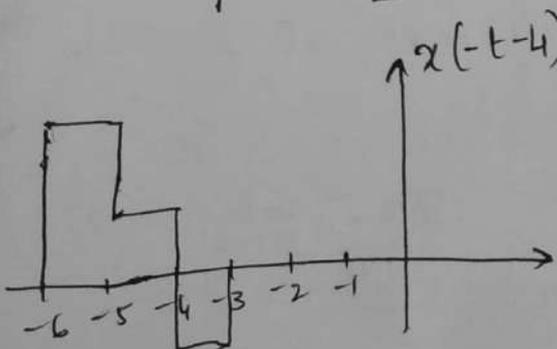


Scale by $-\frac{1}{2}$



or

\Rightarrow



Elementary Signals

↳ Exponential Signals

An exponential signal is represented by

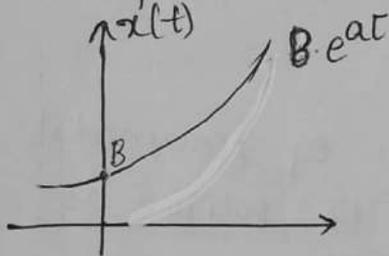
$$x(t) = B \cdot e^{at}$$

where B & a are real parameters

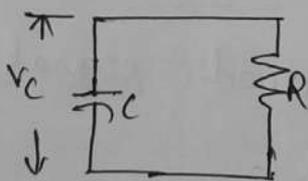
The parameter B is the amplitude of the exponential signal measured at $t=0$. The behaviour of the signal is depends on the parameter 'a' and is of two types -

- ① Growing exponential and
- ② Decaying exponential.

If a is +ve i.e. $a > 0$ then the signal $x(t)$ is called as Growing exponential. This form is used in describing many physical processes including chain reactions in atomic explosions and complex chemical reactions.



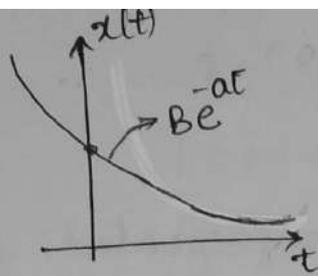
If a is -ve i.e. $a < 0$ then the signal $x(t)$ is called as Decaying exponential. The physical example for this kind of signal is RC circuit.



If C is initially charged at $t=0$ then

$$V_C(t) = V_0$$

Then, $V_C(t)$ decreases exponentially as t increases in $V_C(t) = V_0 e^{-t/RC}$ where RC is the time constant.



In case of Discrete time signal

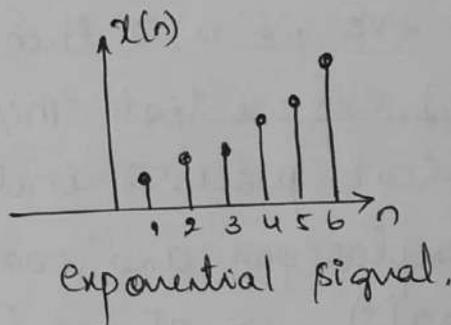
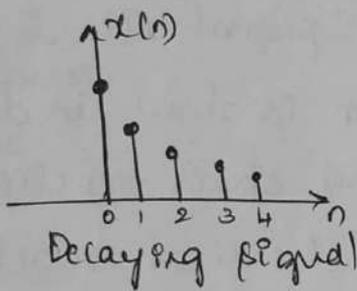
$$x[n] = B \cdot r^n \quad \text{where } r = e^{\alpha}$$

If $0 < r < 1$, then the signal $x(n)$ is Decaying signal

If $r > 1$, then the signal $x(n)$ is growing signal.

If $r < 0$, then the discrete time exponential signal $x(n)$ assumes alternating signs for r^n and is +ve for n even and negative for $n = \text{odd}$.

ex:



27. Sinusoidal Signal.

The Continuous time version of sinusoidal signal in its most general form may be written as

$$x(t) = A \cos(\omega t + \phi) \quad \& \quad x(t) = A \sin(\omega t + \phi)$$

$A \rightarrow$ Amplitude

$\omega \rightarrow$ angular frequency in radians/sec.

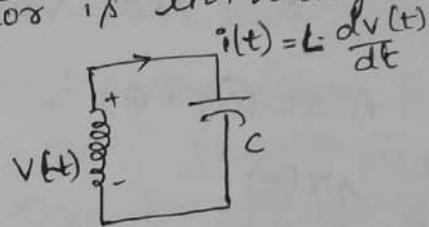
$\phi \rightarrow$ phase angle in radians

A Sinusoidal signal is example of periodic signal with $T = \frac{2\pi}{\omega}$ i.e. $x(t) = x(t+T)$.

$$\begin{aligned}
 \text{Proof: } x(t+T) &= A \cos[\omega(t+T) + \phi] \\
 &= A \cos[\omega t + \omega T + \phi] \\
 &= A \cos[\omega t + \omega \left[\frac{2\pi}{\omega}\right] + \phi] \\
 &= A \cos[\omega t + 2\pi + \phi] \\
 &= A \cos[\omega t + \phi]
 \end{aligned}$$

$$\therefore x(t+T) = x(t)$$

A physical example of generating sinusoidal signal is ideal LC ckt, in which capacitor is initially charged at $t=0$ with voltage across capacitor V_0



Discrete time version of a sinusoidal signal

The period of a periodic discrete time signal is measured in samples. Thus $x(n)$ is said to be periodic with a period of 'N' samples.

$$\text{i.e. } x(n) = A \cos[\Omega n + \phi]$$

$$\text{for periodic, } x(n+N) = x(n)$$

$$\therefore x(n) = A \cos[\Omega n + \Omega N + \phi]$$

only if $\Omega N = 2\pi m$ radians

$$\text{i.e. } N = \frac{2\pi m}{\Omega} \text{ samples}$$

where m, N both are integers

As in continuous time signals, this is not periodic for any arbitrary value of Ω . For the discrete time signals to be periodic, the angular frequency must be integer multiple of 2π .

3) Exponential Damped Sinusoidal Signals.

This signal is obtained as a result of multiplication of sinusoidal signal by a real-valued decaying exponential signal.

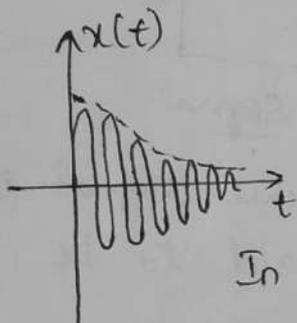
$$\text{i.e. } \boxed{x(t) = A e^{-\alpha t} \sin(\omega t + \phi)}, \quad \alpha > 0$$

Where,

$x(t)$ = Exponential Damped sinusoidal signal

$e^{-\alpha t}$ = real valued decaying exponential signal.

$A \sin(\omega t + \phi)$ = sinusoidal signal



ex: RLC circuit

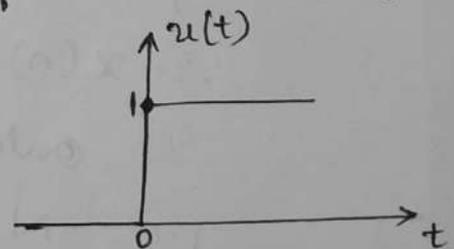
In case of Discrete time signal

$$x[n] = B \alpha^n \sin(\omega n + \phi) \quad \text{if } 0 < |\alpha| < 1$$

4) Unit-Step Function.

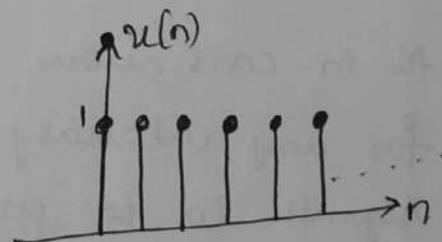
The continuous time version of the unit step function is defined by

$$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



The Discrete time version of the unit step function is defined by

$$u(n) = \begin{cases} 1 & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$



A step function is said to exhibit a discontinuity at t_0 , since the value of $u(t)$ change

12
instantaneously from 0 to 1, at $t=0$.

The simple example is a DC source applied at $t=0$ by closing a switch.

⇒ Impulse & Delta functions

Impulse signal is one of the most important signal and play an important role in the analysis. Unit impulse can be regarded as a rectangular pulse with a width that has become infinitesimal, i.e. extremely small, and height that has become infinitely large and overall area remains unity. Hence unit impulse signal is a signal with zero amplitude everywhere except at $t=0$, and at $t=0$ the amplitude is infinite such that the area under the curve is equal to one.

Mathematically, the continuous time version of the unit impulse signal is defined as,

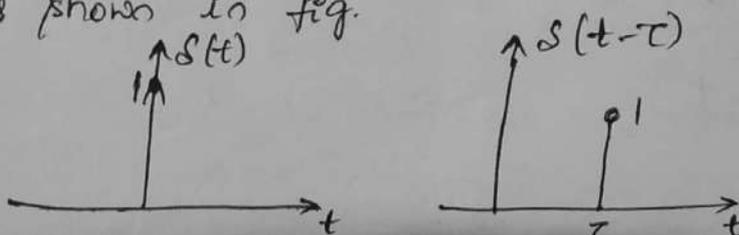
$$S(t) = \begin{cases} 0, & \text{for } t \neq 0 \\ 1, & \text{for } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} S(t) \cdot dt = 1 \Rightarrow \text{Area under } S(t) = 1$$

Delayed unit impulse signal is defined as

$$\int_{-\infty}^{\infty} S(t-\tau) = 1, \quad S(t-\tau) = 0, \quad t \neq \tau$$

The Unit impulse and delayed unit impulse signal is as shown in fig.



$\delta(t)$ is also referred as the Dirac-Delta function.

$\delta(t)$ is also obtained by the derivative of the step function, $u(t)$ and denoted as $\frac{d u(t)}{dt} = \delta(t)$

The Discrete time version of unit impulse is commonly denoted by $\delta[n]$

$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

$$\delta(n) = u(n) - u(n-1)$$

6) Ramp Function

The continuous time version of the ramp function is

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad \& \quad r(t) = t \cdot u(t)$$

The impulse function $\delta(t)$ is the derivative of the step function $u(t)$ w.r.t time. The integral of the step function $u(t)$ is the ramp function of the unit slope.

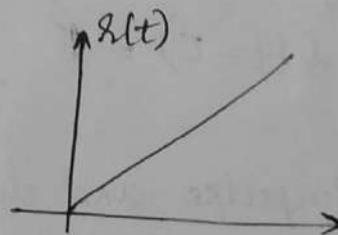
The physical example is a constant current flowing through a capacitor leads to a ramp voltage across capacitor

$$V_c(t) = \frac{1}{c} \int i \cdot dt$$

$$= \frac{I}{c} \int dt$$

$$= \frac{I}{c} t$$

$$\boxed{V_c(t) \propto t}$$

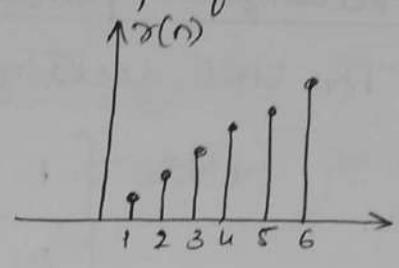


The discrete time version of the ramp function is defined by

$$x[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

or

$$x[n] = n \cdot u[n]$$



⇒ Complex Exponential Signal

The complex exponential signal is defined as,

$$x(t) = A \cdot e^{j\omega_0 t}$$

where, $A \rightarrow$ Amplitude

$\omega_0 \rightarrow$ Angular frequency

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T}$$

Complex exponential function can be resolved into real and imaginary parts i.e

$$\begin{aligned} x(t) &= A \cdot e^{j\omega_0 t} \\ &= A [\cos \omega_0 t + j \sin \omega_0 t] \end{aligned}$$

Exponential sinusoidal signal is defined as

$$x(t) = A e^{at} \sin \omega_0 t$$

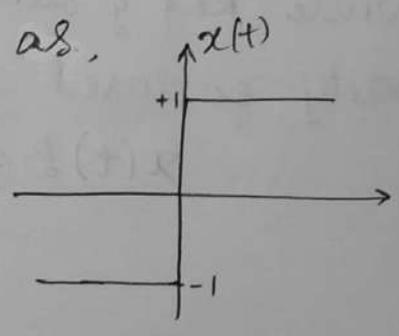
The exponential Discrete time signal is defined by

$$e[n] = a^n \cdot x[n]$$

8) Signum Signal:

Signum signal is defined as,

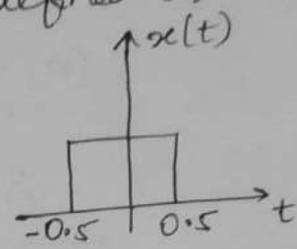
$$x(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$



9) Unit rectangular function

The unit rectangular function is defined as

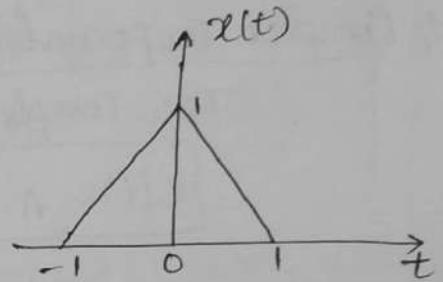
$$x(t) = \begin{cases} 1 & |t| < 0.5 \\ 0 & \text{otherwise} \end{cases}$$



10) Unit triangular function

It is defined as

$$x(t) = \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$



Relation between Sinusoidal And Complex Exponential Signals.

Formula

Consider the complex exponential $e^{j\theta}$ using Euler's

$$e^{j\theta} = \cos\theta + j\sin\theta \rightarrow \textcircled{1}$$

eq-① indicates that we can express the continuous time sinusoidal signal,

$$x(t) = A \cos(\omega t + \phi)$$

as the real part of the complex exponential signal

$$B e^{j\omega t} = x(t) \rightarrow \textcircled{2}$$

$$\text{where } B = A e^{j\phi} \rightarrow \textcircled{3}$$

$$\therefore A \cos(\omega t + \phi) = \text{Re}(B e^{j\omega t}) \rightarrow \textcircled{4}$$

where $\text{Re}\{z\}$ denotes the real part of the complex quantity enclosed inside the braces.

$$B e^{j\omega t} = A e^{j\phi} \cdot e^{j\omega t}$$

$$= A e^{j(\omega t + \phi)}$$

$$= A [\cos(\omega t + \phi) + j \sin(\omega t + \phi)]$$

$$= A \cos(\omega t + \phi) + A j \sin(\omega t + \phi) \rightarrow \textcircled{5}$$

A continuous time sinusoidal signal in terms of a sine function

$$x(t) = A \sin(\omega t + \phi) \rightarrow \textcircled{1}$$

which is represented by the imaginary part of the complex exponential signal $Be^{j\omega t}$. That is

$$A \sin(\omega t + \phi) = \text{Im}\{Be^{j\omega t}\} \rightarrow \textcircled{2}$$

where B is defined in eq-3 $\text{Im}\{\}$ denotes the imaginary part of the complex quantity enclosed inside the braces.

→ The sinusoidal signal $A \cos(\omega t + \phi)$ lags behind the sinusoidal signal $A \sin(\omega t + \phi)$ for $\phi = \pi/6$.

||| In case of Discrete Time,

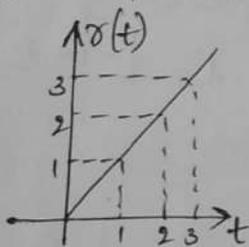
$$A \cos(\Omega n + \phi) = \text{Re}\{Be^{j\Omega n}\}$$

$$A \sin(\Omega n + \phi) = \text{Im}\{Be^{j\Omega n}\}$$

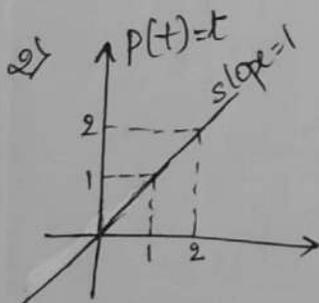
where B is defined in terms of A & ϕ .

Note:

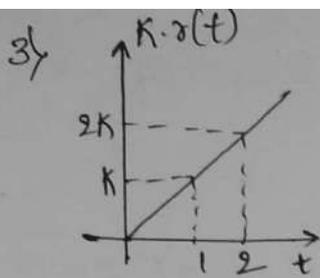
1) Ramp signal with unit slope



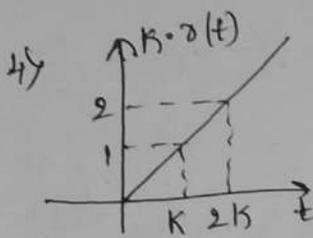
$$\text{slope } m = \frac{y}{x} = \frac{1}{1} = 1$$



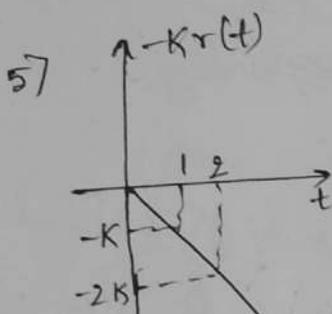
$p(t)$ is a function passing through $t=0$ existing from $-\infty$ to ∞ .



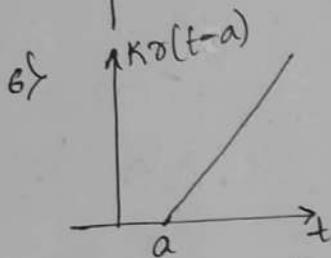
Signal $k \cdot r(t)$ is a ramp of slope k starting at $t=0$, where $k > 1$



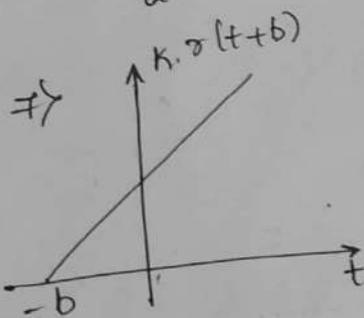
Here, signal $k \cdot r(t)$ is a ramp of slope $1/k$ starting at $t=0$ where $k < 1$



Signal $-k \cdot r(t)$ is a ramp of slope $-k$



$k \cdot r(t-a)$ is a delayed ramp shifted by a units, and k is a slope, and signal starts at $t=a$ and extending over $t = (a, \infty)$



$k \cdot r(t+b)$ is a delayed ramp by $-b$ units and k is a slope, and the signal starts at $t=-b$ and extending in the region $t = (-b, \infty)$.

8) $k \cdot r(b-t)$ is a ramp of slope k , starting at $t=b$ and extending in the region $t = [-\infty, b]$, is a left sided signal.

9) The differentiation of the ramp signal is a unit step function.

$$\frac{d r(t)}{dt} = u(t)$$

10) Differentiation of step function gives an impulse 15

$$\frac{d u(t)}{dt} = \delta(t).$$

By Repeated integration, we can get original signal.

11) The ramp signal can be used as the building block. point to point addition of two ramps of opposite slopes produces step function

12) Step \times ramp = ramp

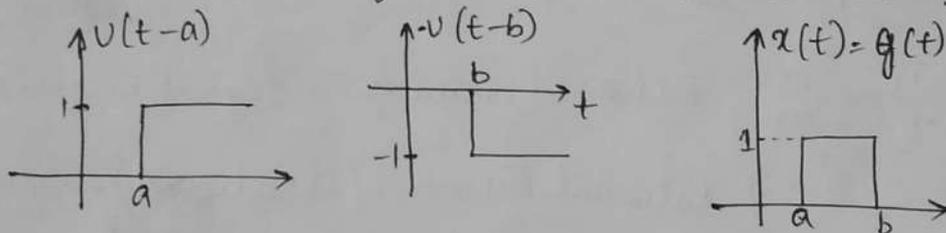
Step + (+ve ramp) = +ve ramp

Step + (-ve ramp) = -ve ramp.

13) Consider a signal as the difference of two step function.

$$x(t) = u(t-a) - u(t-b) \quad b > a.$$

Resultant signal $x(t)$ is called as "Gate function"



$g(t)$ has value unity b/w $t=a$ & $t=b$. Thus.

$$g(t) = m \cdot \text{rect} \left[\frac{t-a}{b} \right]$$

Represents a gate signal symmetric about $t=a$ and width b and height m .

ex: $5 \text{rect} \left[\frac{t+3}{2} \right]$ is a gate function centered at -3 of width 2 and height 5.

Problems:

Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 & T_2 respectively. Under what condition is the sum $x(t) = x_1(t) + x_2(t)$ be periodic and what is the fundamental period of it. If it is periodic.

Soln: $x_1(t) = x_1(t+T_1) = x_1(t+mT_1)$, $m = +ve$ integer
 $x_2(t) = x_2(t+T_2) = x_2(t+nT_2)$, $n = +ve$ integer

$$x(t) = x_1(t+mT_1) + x_2(t+nT_2) \rightarrow \textcircled{1}$$

Also

$x(t) = x(t+T)$ if $x(t)$ is periodic

then

$$x(t+T) = x_1(t+T) + x_2(t+T) \rightarrow \textcircled{2}$$

comparing $\textcircled{1}$ & $\textcircled{2}$

$$mT_1 = nT_2$$

$\therefore \frac{T_1}{T_2} = \frac{n}{m}$ = Rational number \rightarrow signal is periodic

\neq Rational number (irrational) - non-periodic

Q) Find the period of the following continuous time signals

$$\rightarrow x(t) = e^{j2\pi t/10}$$

Given $x(t)$ is periodic complex exponential signal and comparing with,

$$x(t) = A e^{j\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{10} \Rightarrow T = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$

$$\boxed{T = 10}$$

2) $x(t) = \cos(t + \pi/2)$

Given $x(t)$ is periodic signal and comparing with

$$x(t) = A \cos(\omega_0 t + \phi)$$

$$\omega_0 = 1 \quad \therefore T = \frac{2\pi}{\omega_0} = 2\pi$$

$$T = 2\pi \text{ seconds}$$

3) $x(t) = j e^{j5t}$

Given $x(t)$ is periodic exponential signal and comparing with

$$x(t) = A e^{j\omega_0 t}$$

$$\omega_0 = 5 \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{5}$$

4) $x(t) = \cos\sqrt{2}t + \sin t$

The signal $x(t)$ is a mixture of two signals $x_1(t)$ & $x_2(t)$

$$x_1(t) = \cos\sqrt{2}t \quad x_2(t) = \sin t$$

Comparing these signals with the standard forms

$$x_1(t) = A \cos(\omega_0 t + \phi) \quad x_2(t) = A \sin(\omega_0 t + \phi)$$

$$\omega_1 = \sqrt{2} \quad \therefore T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{2}} \text{ sec}$$

$$\omega_2 = 1 \quad \therefore T_2 = \frac{2\pi}{\omega_2} = 2\pi \text{ sec}$$

$$\frac{T_1}{T_2} = \frac{2\pi/\sqrt{2}}{2\pi} = \frac{1}{\sqrt{2}}$$

$\Rightarrow \frac{T_1}{T_2}$ is not a rational number hence $x(t)$ is a non-periodic

5) $x(t) = 5 \cos \frac{2\pi}{5}t + 7 \cos \frac{2\pi}{3}t$

$$x_1(t) = 5 \cos \frac{2\pi}{5}t$$

$$x_2(t) = 7 \cos \frac{2\pi}{3}t$$

Comparing with the standard signal,

$$x(t) = A \cos(\omega_0 t + \phi)$$

$$\omega_1 = \frac{2\pi}{5} \Rightarrow T = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/5} = 5 \text{ sec}$$

$$\omega_2 = \frac{2\pi}{3} \Rightarrow T = \frac{2\pi}{\omega_2} = \frac{2\pi}{2\pi/3} = 3 \text{ sec}$$

$$\therefore \frac{T_1}{T_2} = \frac{5}{3}$$

Since T_1/T_2 is a rational number hence the given $x(t)$ is periodic

The fundamental period of the $x(t)$ is the LCM of T_1 & T_2 . Hence the period of the signal $x(t)$ is 15 seconds.

$$8) x(t) = 2 \sin^2 t = (1 - \cos 2t)$$

The first term is DC term. Second term is a sinusoidal signal and is always periodic signal.

Comparing with $x(t) = A \cos(\omega_0 t + \phi)$

$$\omega_0 = 2 \therefore T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi \text{ seconds.}$$

$\Rightarrow x(t)$ is periodic with period π seconds.

$$9) x(t) = [\sin(t - \pi/6)]^2$$

$$x(t) = \frac{1}{2} [1 - \cos 2(t - \pi/6)]$$

$$= \frac{1}{2} - \frac{1}{2} \cos 2(t - \pi/6)$$

$$= \frac{1}{2} - \frac{1}{4} [\cos 2t \cos \pi/3 + \sin 2t \sin \pi/3]$$

$$= \frac{1}{2} - \frac{1}{4} \cos 2t - \frac{\sqrt{3}}{4} \sin 2t$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$$

$$= \frac{1}{2} - \frac{1}{4} \cos 2t - \frac{\sqrt{3}}{4} \sin 2t$$

The first term is DC term. The second and third term is mixture of two signals.

Comparing with the standard signals

$$x(t) = A \cos(\omega_0 t + \phi) \quad \text{and} \quad x(t) = A \sin(\omega_0 t + \phi)$$

$$\omega_1 = 2 \quad , \quad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2} = \pi \text{ sec}$$

$$\omega_2 = 2 \quad , \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{2} = \pi \text{ sec}$$

$\therefore \frac{T_1}{T_2} = \frac{\pi}{\pi}$. Since T_1/T_2 is a rational number.

hence $x(t)$ is periodic. LCM of T_1 & T_2 is π .

$\therefore x(t)$ is periodic with period π sec.

$$\text{8) } x(n) = 5 \sin 2n$$

$$\Omega = 2 \text{ radians}$$

$$2\pi f = 2$$

$$f = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$N = \pi$$

$\therefore x(n)$ is a non-periodic

$$\text{9) } x(n) = \sin \left[\frac{1}{5} \pi n \right] \cdot \sin \left[\frac{1}{3} \pi n \right]$$

$$\therefore x(n) = \frac{1}{2} \left[\cos \frac{2\pi}{15} n - \cos \frac{8\pi}{15} n \right]$$

$$\sin A \sin B = \frac{1}{2} \left[\cos(A-B) - \cos(A+B) \right]$$

$$\text{let } \Omega_1 = \frac{8\pi}{15} \quad \text{and} \quad \Omega_2 = \frac{2\pi}{15}$$

$$\Omega_1 = \frac{2\pi}{N} = \frac{2\pi}{\frac{8\pi}{15}} = 15 \text{ samples}$$

$$\omega_2 = \frac{2\pi}{N_2} = \frac{2\pi}{2\pi/15}$$

$$N_2 = 15 \text{ samples.}$$

$$\frac{N_1}{N_2} = \frac{15}{15} = \text{Rational number}$$

$$\text{LCM of } N_1 \text{ \& } N_2 = 15 \text{ samples.}$$

\therefore period of the total signal = 15 samples.

Calculate Energy and power for the following C.T.S and classify it as energy or power signal.

$$\text{i) } x(t) = 5 \cos 5\omega t$$

The energy of the C.T.S is,

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |5 \cos 5\omega t|^2 dt = \lim_{T \rightarrow \infty} \int_{-T}^T 25 \cos^2 5\omega t dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T 25 \left[\frac{1 + \cos 10\omega t}{2} \right] dt = \lim_{T \rightarrow \infty} \frac{25}{2} \left[t + \frac{\sin 10\omega t}{10\omega} \right]_{-T}^T$$

$$= \lim_{T \rightarrow \infty} \frac{25}{2} \left[t + \frac{\sin 10 \cdot 2\pi/f \cdot t}{10 \cdot 2\pi/f} \right]_{-T}^T = \lim_{T \rightarrow \infty} \frac{25}{2} [2T]$$

$$= \lim_{T \rightarrow \infty} [25T] \Rightarrow \boxed{E = \infty}$$

The power P of a C.T.S is

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |5 \cos 5\omega t|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T 25(10)^2 \cos^2 \omega t \cdot dt = \lim_{T \rightarrow \infty} \int_{-T}^T 25 \left[\frac{1 + \cos(10\omega t)}{2} \right] \cdot dt$$

$$= \lim_{T \rightarrow \infty} \frac{25}{4T} \left[t + \frac{\sin(10\omega t)}{10\omega} \right]_{-T}^T = \lim_{T \rightarrow \infty} \frac{25}{4T} \left[t + \frac{\sin(10 \cdot 2\pi/t \cdot T)}{10 \cdot 2\pi/t} \right]_{-T}^T$$

$$= \lim_{T \rightarrow \infty} \frac{25}{4T} [2T] = 12.5 \text{ W}$$

$$\therefore \boxed{P = 12.5 \text{ W}}$$

\therefore The Given signal is a power signal.

2) $e^{-a|t|}$, $a > 0$

$$\text{Given } x(t) = e^{-a|t|} = \begin{cases} e^{-at} & \text{when } t > 0 \\ e^{at} & \text{when } t < 0 \end{cases}$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 \cdot dt = \int_{-\infty}^{\infty} e^{-2a|t|} \cdot dt \Rightarrow \int_{-\infty}^0 e^{2at} \cdot dt + \int_0^{\infty} e^{-2at} \cdot dt$$

$$\boxed{E = \frac{1}{a} \text{ joules}}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 \cdot dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |e^{-a|t|}|^2 \cdot dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2Ta} = 0 \text{ W}$$

$$\therefore \boxed{P = 0 \text{ W}}$$

The Given signal is a Energy signal.

3) $x(t) = A \cos(\omega t + \theta)$

The Energy of the LTS is,

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 \cdot dt = \lim_{T \rightarrow \infty} \int_{-T}^T |A \cos(\omega t + \theta)|^2 \cdot dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T \frac{A^2}{2} [1 + \cos(2\omega t + 2\theta)] \cdot dt$$

$$= \infty$$

$$\therefore E = \infty$$

The power of the C.T.S is

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 \cdot dt$$

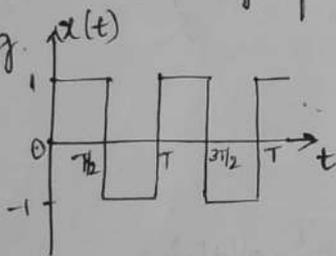
$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T A^2 \cos^2(\omega t + \theta) \cdot dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \frac{A^2}{2} (1 + \cos(2\omega t + 2\theta)) dt$$

$$P = \frac{A^2}{2} \omega$$

Hence the given signal $x(t) = A \cos(\omega t + \theta)$ is a power signal.

4) Find the average power of the square wave shown in the fig.



$$\text{Soln: } x(t) = \begin{cases} 1 & \text{for } 0 < t < T/2 \\ -1 & \text{for } T/2 < t < T \end{cases}$$

In a periodic signal, avg power is defined for one cycle

\therefore Avg power is given as,

$$P = \frac{1}{T} \int_0^T |x(t)|^2 dt.$$

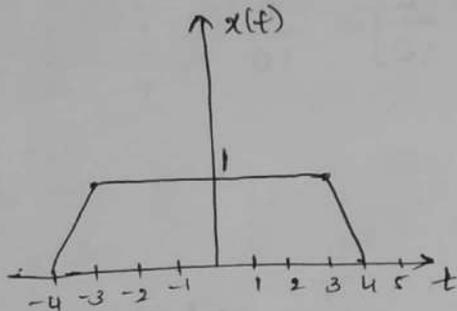
$$P = \frac{1}{T} \int_0^{T/2} 1^2 \cdot dt + \frac{1}{T} \int_{T/2}^T (-1)^2 dt$$

$$= \frac{1}{T} \left[(t)_{0}^{T/2} + (t)_{T/2}^T \right]$$

$$= \frac{1}{T} \left[\frac{T}{2} - 0 + T - \frac{T}{2} \right] = 1W.$$

$$\therefore \boxed{P = 1W}$$

6) Find the total energy of the Trapezoidal pulse.



$$\Rightarrow x(t) = \begin{cases} t+4, & -4 \leq t \leq -3 \\ 1, & -3 \leq t \leq 3 \\ 4-t, & 3 \leq t \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Energy } E = \int_{-4}^{+4} |x(t)|^2 dt$$

$$= \int_{-4}^{-3} (t+4)^2 dt + \int_{-3}^3 1^2 dt + \int_3^4 (4-t)^2 dt$$

$$E = \left. \frac{(t+4)^3}{3} \right|_{-4}^{-3} + \left. t \right|_{-3}^3 + \left. \frac{(4-t)^3}{3} \right|_3^4$$

$$E = \frac{1}{3} + 6 + \frac{1}{3} = \frac{20}{3} \text{ Joules.}$$

$$\boxed{E = 6.675J}$$

$$6) x(t) = e^{-\alpha t} \cdot u(t)$$

The energy E of a C.T.S is given by

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \int_{-T}^T |e^{-\alpha t} \cdot u(t)|^2 dt$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |e^{-5t}|^2 dt = \lim_{T \rightarrow \infty} \int_{-T}^T |e^{-10t}| dt$$

$$= \lim_{T \rightarrow \infty} \left[\frac{e^{-10t}}{-10} \right]_0^T$$

$$\lim_{T \rightarrow \infty} \left[\frac{e^{-10T}}{-10} - \frac{e^0}{-10} \right] = \lim_{T \rightarrow \infty} \left[\frac{1}{10} - \frac{e^{-10T}}{10} \right]$$

$$= \left[\frac{1}{10} - \frac{0}{10} \right] = \frac{1}{10}$$

$$\therefore \boxed{E = 1/10 \text{ J}}$$

The power of a C.T.S is

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T |e^{-5t}|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T |e^{-10t}| dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\frac{e^{-10t}}{-10} \right]_0^T$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\frac{e^{-10T}}{-10} - \frac{e^0}{-10} \right] = \frac{1}{\infty} \left[\frac{-0}{10} + \frac{1}{10} \right] = 0$$

$$\boxed{P = 0 \text{ Watts}}$$

⇒ $x(t) = t \cdot u(t)$ - find energy and power.

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_0^T t^2 dt$$

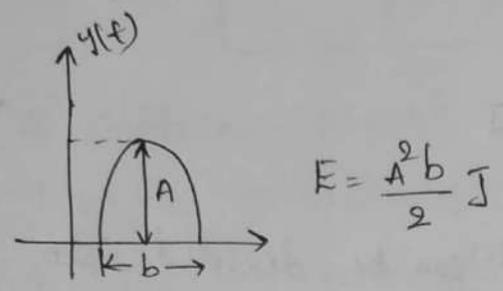
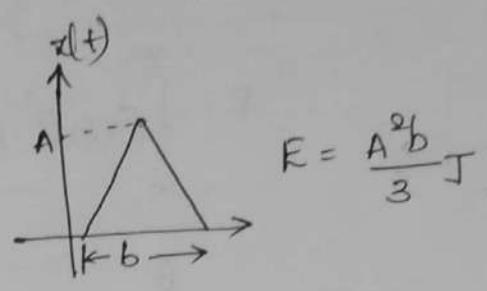
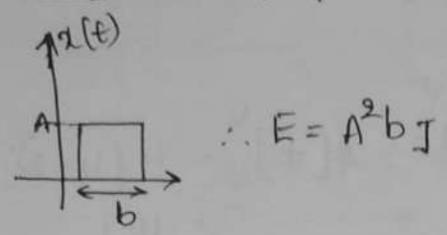
$$t \cdot u(t) = \begin{cases} t, & \text{when } t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\boxed{E = \infty}$$

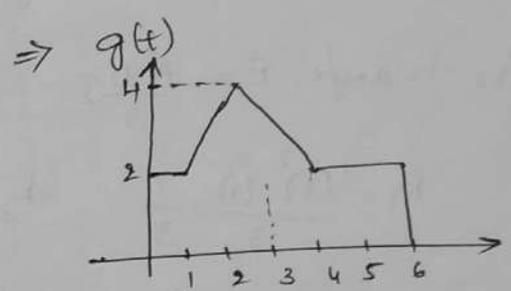
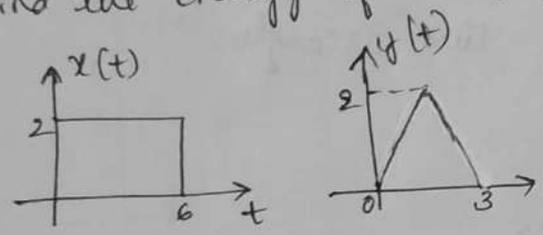
$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T t^2 dt = \infty$$

$\boxed{P = \infty}$ Hence, the ramp signal is neither energy nor a power signal.

Note :: The signal energy for the useful shapes are.



8) Find the energy of the signals. $g(t) = x(t) + y(t)$, where.



$$E_{g(t)} = \int_0^6 [x(t) + y(t)]^2 dt$$

$$= \int_0^6 x^2(t) + y^2(t) + 2x(t) \cdot y(t) dt$$

$$= E_x(t) + E_y(t) + E_{xy}(t)$$

$$= 24 \text{ J} + 4 \text{ J} + 16 \text{ J}$$

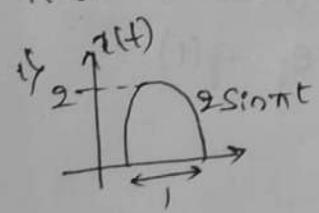
$$E_x(t) = A^2 b = 2^2 \times 6 = 24 \text{ J}$$

$$E_y(t) = \frac{A^2 b}{3} = \frac{2^2 \times 3}{3} = 4 \text{ J}$$

$$E_{xy}(t) = \frac{1}{3} (4)^2 \cdot 3 = 16 \text{ J}$$

$E_{g(t)} = 60 \text{ J}$

9) Find the energy of the following signals shown.

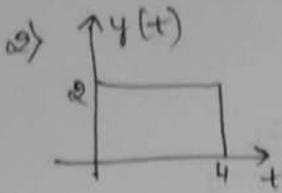


$$E = \int_{-\infty}^{\infty} (2 \sin \pi t)^2 dt = \int_0^1 4 \cdot \frac{1 - \cos 2\pi t}{2} dt$$

$$E = 2 \int_0^1 (1 - \cos 2\pi t) dt = 2 \left[t \Big|_0^1 - \frac{\sin 2\pi t}{2\pi} \Big|_0^1 \right]$$

$$E = 2 [1] = 2 \text{ J}$$

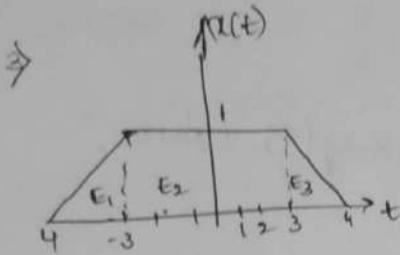
Using the formula, $E = \frac{A^2 b}{2} = \frac{2^2 \cdot 1}{2} = 2 \text{ J}$



$$E = \int_{-\infty}^{\infty} [x(t)]^2 \cdot dt = \int_0^4 (2)^2 \cdot dt = 4 [t]_0^4 = 4 [4 - 0] = 16 \text{ J}$$

⊙

$$E = A^2 b = 2^2 \cdot 4 = 16 \text{ J}$$



The given signal can be divided into three signals E_1, E_2, E_3 i.e. are rectangle and two triangles.

for triangle, $E = \frac{A^2 b}{3}$

$$E_1 = \frac{(1)^2 \cdot (1)}{3} = \frac{1}{3} \quad E_3 = \frac{(1)^2 \cdot (1)}{3} = \frac{1}{3}$$

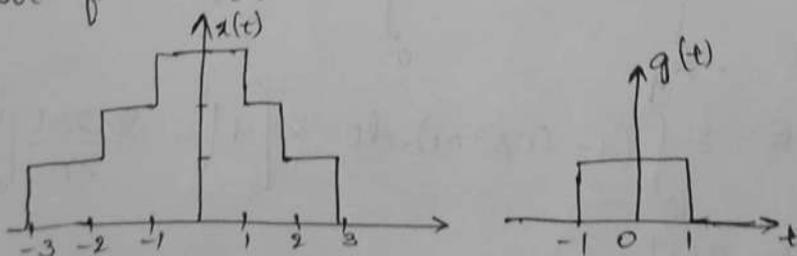
$$E_2 = A^2 b = (1)^2 \cdot (6) = 6$$

$$\therefore E = E_1 + E_2 + E_3$$

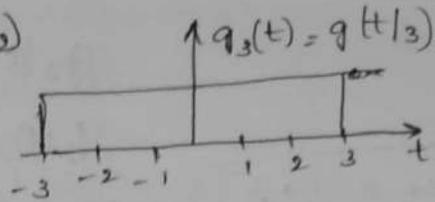
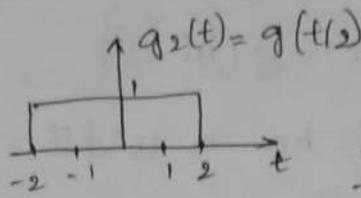
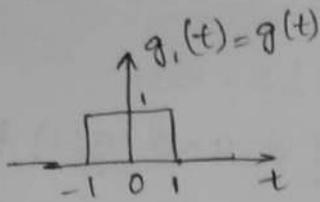
$$= \frac{1}{3} + 6 + \frac{1}{3} = \frac{1 + 18 + 1}{3}$$

$$E = \frac{20}{3} \text{ J}$$

10) Fig shows is a pulse $x(t)$, that may be viewed as the superposition of three rectangular pulses. Using rectangular pulse $g(t)$ shown as a building block, construct the waveform $x(t)$ and express $x(t)$ in terms of $g(t)$.



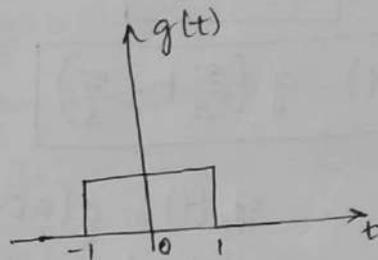
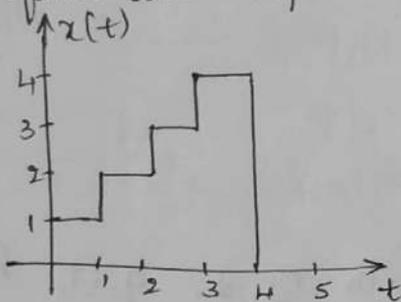
Soln:



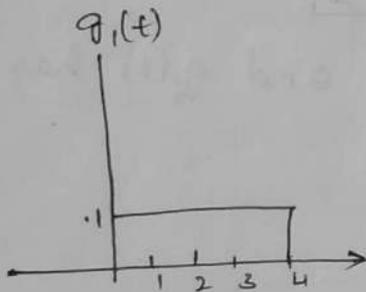
$$\therefore x(t) = g_1(t) + g_2(t) + g_3(t)$$

$$\boxed{x(t) = g(t) + g(t/2) + g(t/3)}$$

1) A staircase signal \$x(t)\$ that may be viewed as superposition of 4 rectangular pulses, starting with the rectangular pulses \$g(t)\$ as shown. Construct these waveforms and express \$x(t)\$ in terms of \$g(t)\$.



Soln:



$$g_1(t) = g(at-b)$$

\$g(t)\$ has a width of 2. \$g_1(t)\$ has a width of 4

\$\therefore\$ it is expanded by 2

$$\Rightarrow \boxed{a = 1/2}$$

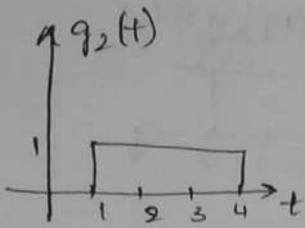
The mid-point of \$g(t)\$ is at \$t = 0\$, whereas the mid-point of \$g_1(t)\$ is at 2. Hence, we must choose 'b' to satisfy the condition.

$$at - b = 0$$

$$\frac{1}{2}(2) - b = 0$$

$$\therefore \boxed{b = 1}$$

$$\therefore \boxed{g_1(t) = g(1/2 t - 1)}$$



$$g_2(t) = g(at-b)$$

$g_2(t)$ has a width of 3, and $g(t)$ has a width of 2

$\therefore g_2(t)$ is expanded by 1.5 times.

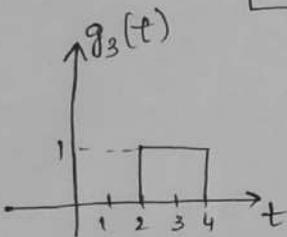
$$\text{i.e. } a = \frac{2}{3}$$

Mid-point of $g_2(t)$ is at $5/2$, whereas $g(t)$ is at $t=0$

$$\therefore a(5/2) - b = 0 \Rightarrow b = \frac{2}{3} \times \frac{5}{2}$$

$$\therefore b = \frac{5}{3}$$

$$\therefore g_2(t) = g\left(\frac{2}{3}t - \frac{5}{3}\right)$$



$$g_3(t) = g(at-b)$$

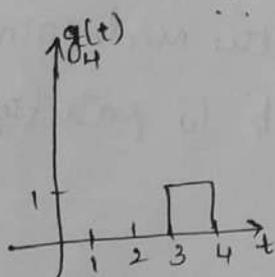
$g(t)$ has a width of 2, $g_3(t)$ has a width of 2. $\therefore a=1$

$g(t)$ has a mid-point at $t=0$, and $g_3(t)$ has a mid-point $t=3$,

$$\therefore at-b=0$$

$$1(3) = b \quad \therefore b=3$$

$$\therefore g_3(t) = g(t-3)$$



$$g_4(t) = g(at-b)$$

$g(t)$ has a width of 2, $g_4(t)$ has a width of 1. \therefore it is expanded by $1/2$ time $\therefore a=2$

$g(t)$ has a mid-point at $t=0$ and $g_4(t)$ has mid-point at $t=3.5$

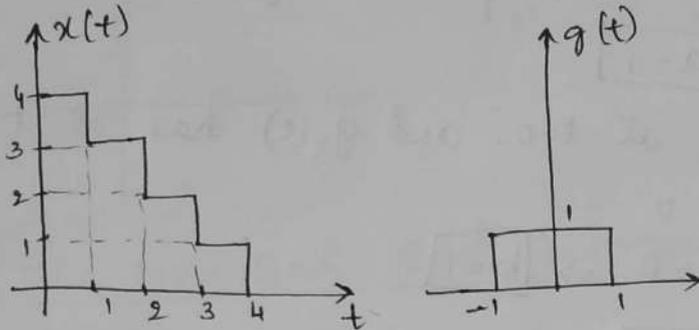
$$\therefore a(7/2) - b = 0$$

$$\therefore b=7 \Rightarrow g_4(t) = g(2t-7)$$

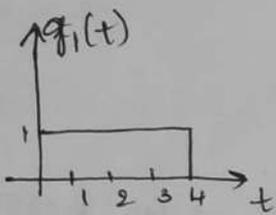
$$\therefore x(t) = g_1(t) + g_2(t) + g_3(t) + g_4(t)$$

$$x(t) = g\left(t/2 - 1\right) + g\left(\frac{2}{3}t - \frac{5}{3}\right) + g(t - 3) + g(2t - 7)$$

1) Express the staircase signal $x(t)$ in terms of $g(t)$



Soln:



$$g_1(t) = g(at - b)$$

$g(t)$ has a width of 2. $g_1(t)$ has a width of 4. \therefore it is expanded by 2 times

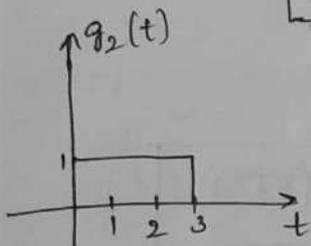
$$\therefore a = 1/2$$

$g(t)$ has a mid-point at $t=0$ and $g_1(t)$ has a mid-point at $t=2$

$$at - b = 0$$

$$\frac{1}{2}(2) - b = 0 \quad \therefore b = 1$$

$$\therefore g_1(t) = g\left(t/2 - 1\right)$$



$$g_2(t) = g(at - b)$$

$g(t)$ has a width of 2. $g_2(t)$ has a width of 3. \therefore it is expanded by $\frac{3}{2}$ times.

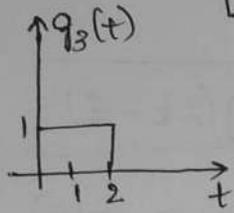
$$\therefore a = 2/3$$

$g(t)$ has a mid-point at $t=0$ and $g_2(t)$ has a mid-point at $t=1.5 = 3/2$

$$\therefore at - b = 0$$

$$\frac{2}{3} \times \frac{3}{2} - b = 0 \quad \therefore b = 1$$

$$\therefore \boxed{g_2(t) = g\left(\frac{2}{3}t - 1\right)}$$



$$g_3(t) = g(at - b)$$

$g(t)$ has a width of 2. $g_3(t)$ has a width of 2. \therefore it is expanded by 1 unit

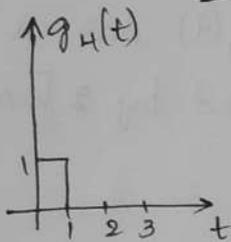
$$\Rightarrow \boxed{a=1}$$

$g(t)$ has a mid-point at $t=0$. and $g_3(t)$ has at $t=1$

$$\therefore at - b = 0$$

$$1(1) - b = 0 \Rightarrow \boxed{b=1}$$

$$\boxed{g_3(t) = g(t - 1)}$$



$$g_4(t) = g(at - b)$$

$g(t)$ has a width of 2. and $g_4(t)$ has a width of 1. \therefore it is compressed by $\frac{1}{2}$ unit

$$\Rightarrow \boxed{a=2}$$

$g(t)$ has a mid-point at $t=0$. $g_4(t)$ has at $t=1/2$

$$a(1/2) - b = 0$$

$$\therefore \boxed{b=1}$$

$$\boxed{g_4(t) = g(2t - 1)}$$

$$x(t) = g_1(t) + g_2(t) + g_3(t) + g_4(t)$$

$$\boxed{x(t) = g\left(t/2 - 1\right) + g\left(\frac{2}{3}t - 1\right) + g(t - 1) + g(2t - 1)}$$

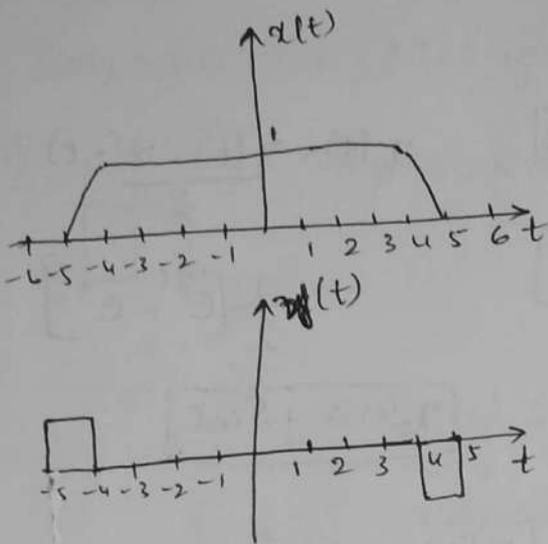
13) A trapezoidal pulse $x(t)$ defined by,

$$x(t) = \begin{cases} 5-t, & 4 \leq t \leq 5 \\ 1, & -4 \leq t \leq 4 \\ t+5, & -5 \leq t \leq -4 \\ 0, & \text{otherwise.} \end{cases}$$

is applied to differentiator having the ip-op relation

$$y(t) = \frac{d}{dt} x(t)$$

Find the energy of the signal.



$$E_{\text{avg}}(t) = \int_{-5}^{-4} (1)^2 dt + \int_{4}^5 (-1)^2 dt$$

$$= t \Big|_{-5}^{-4} + t \Big|_{4}^5$$

$$E = 1 + 1 = 2 \text{ J}$$

$$E = 2 \text{ J}$$

14) Find the even and odd components of the following signals.

a) $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$
 sol: $x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$

w.k.t, $x_e(t) = \frac{x(t) + x(-t)}{2}$

$$= \frac{1}{2} [2(1 + 3t^2 + 9t^4)]$$

$$x_e(t) = 1 + 3t^2 + 9t^4$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{1}{2} [2(t + 5t^3)]$$

$$x_o(t) = t + 5t^3$$

b) $x(t) = 1 + t \cos t + t^2 \sin t + t^3 \sin t \cos t$

$$x(-t) = 1 - t \cos t - t^2 \sin t + t^3 \sin t \cos t$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{1}{2} [2(1 + t^3 \sin t \cos t)]$$

$$x_e(t) = 1 + t^3 \sin t \cos t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{1}{2} [2(t \cos t + t^2 \sin t)]$$

$$x_o(t) = t \cos t + t^2 \sin t$$

$$c) x(t) = e^{jt}$$

$$x(-t) = e^{-jt}, \quad x_e(t) = \frac{x(t) + x(-t)}{2} \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{1}{2} [e^{jt} + e^{-jt}] \quad = \frac{1}{2} [e^{jt} - e^{-jt}]$$

$$\boxed{x_e(t) = \cos t}$$

$$\boxed{x_o(t) = j \sin t}$$

$$d) x(t) = e^{-2t} \cos t$$

$$x(-t) = e^{-2(-t)} \cdot \cos(-t)$$

$$= e^{2t} \cos t$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

$$= \frac{1}{2} [e^{-2t} \cos t + e^{2t} \cos t]$$

$$= \frac{1}{2} \cos t [e^{-2t} + e^{2t}]$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

$$= \frac{1}{2} [e^{-2t} \cos t - e^{2t} \cos t]$$

$$= \frac{1}{2} \cos t [e^{-2t} - e^{2t}]$$

$$\boxed{x_e(t) = \cosh(2t) \cdot \cos t}$$

$$\boxed{x_o(t) = -\sinh(2t) \cdot \cos t}$$

$$e) x(t) = \cos t + \sin t + \sin t \cdot \cos t$$

$$x(-t) = \cos t - \sin t - \sin t \cos t$$

$$x_e(t) = \frac{2 \cos t}{2} = \cos t$$

$$x_o(t) = \frac{2(\sin t + \sin t \cos t)}{2}$$

$$= \sin t (1 + \cos t)$$

$$f) x(t) = (1+t^3) \cos^3(10t) = \cos^3 10t + t^3 \cos^3 10t$$

$$x(-t) = (1-t^3) \cos^3(10t) = \cos^3 10t - t^3 \cos^3 10t$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

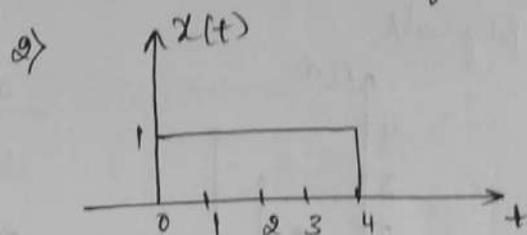
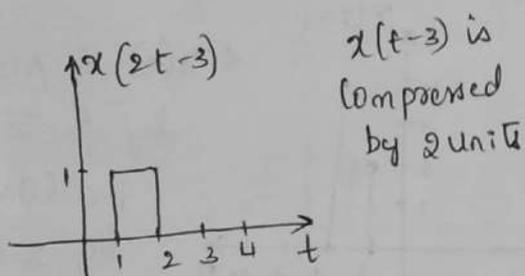
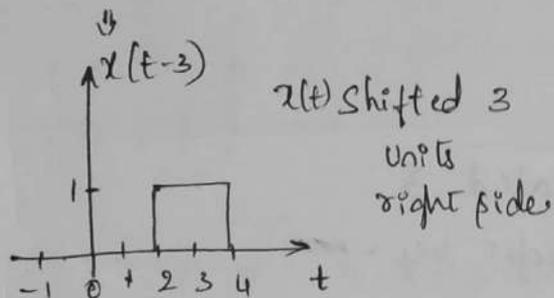
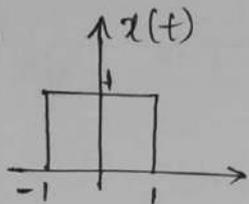
$$= \frac{2 \cos^3 10t}{2} = \cos^3 10t$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

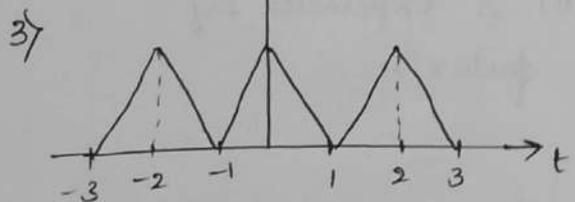
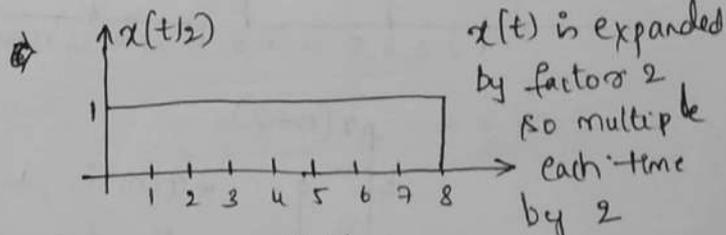
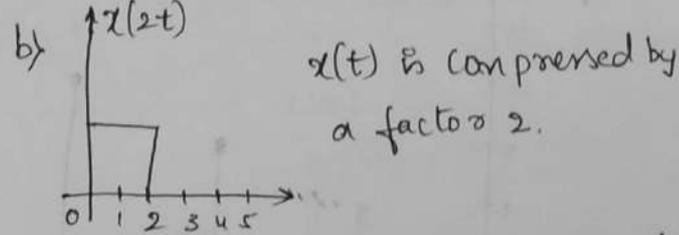
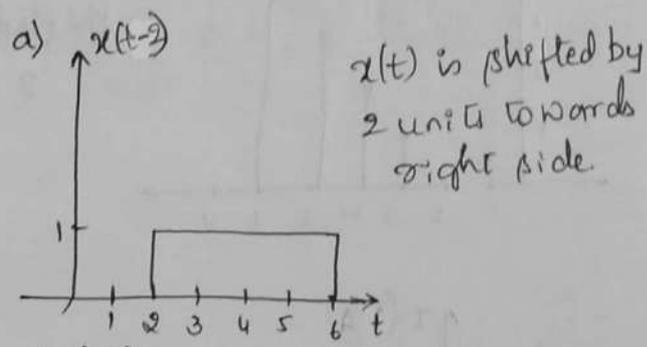
$$= \frac{2t^3 \cos^3 10t}{2} = t^3 \cos^3 10t$$

15) Consider the C.T.S given. Sketch and label the following

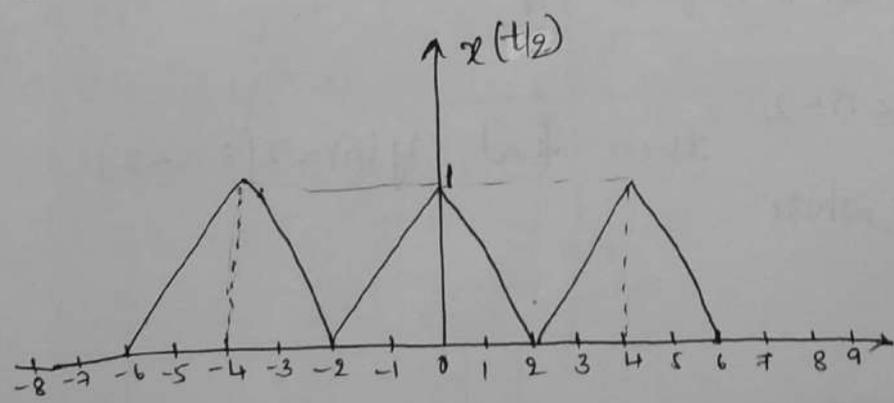
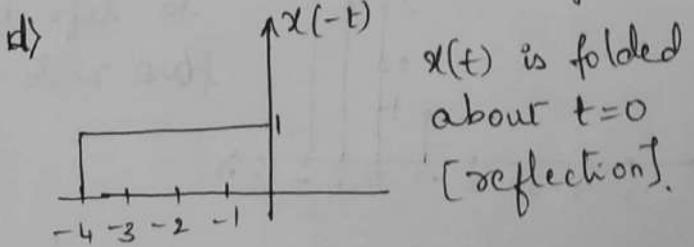
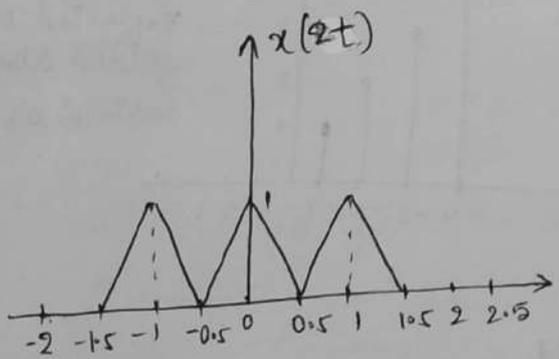
1) find $x(2t-3)$



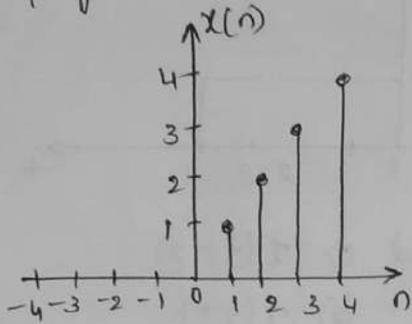
find a) $x(t-2)$
 b) $x(2t)$
 c) $x(t/2)$ d) $x(-t)$



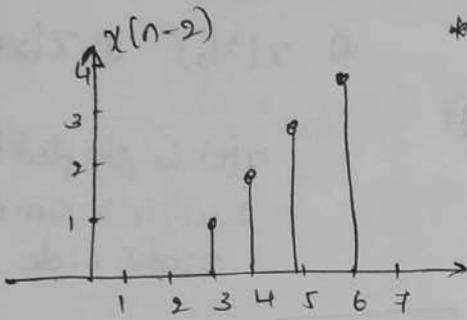
find $x(t/2)$ and $x(2t)$



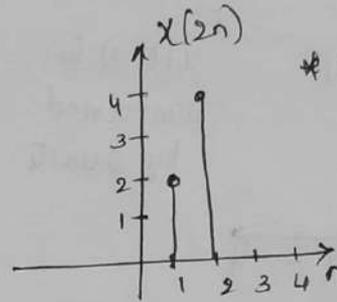
16) A D.T.S $x(n]$ is as shown in fig. sketch the following signals.



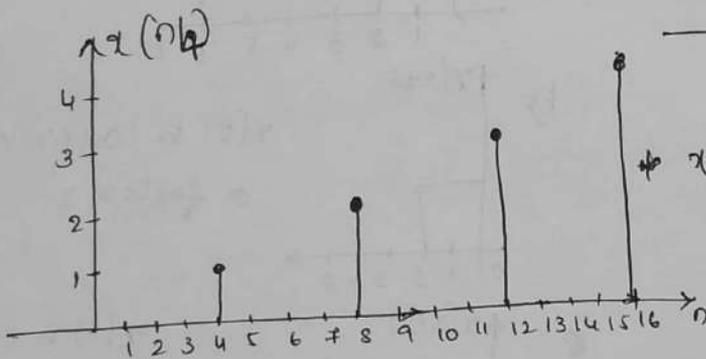
- a) $x[n-2]$
- b) $x[2n]$
- c) $x[n/4]$
- d) $x[n+2]$
- e) $x[-n]$



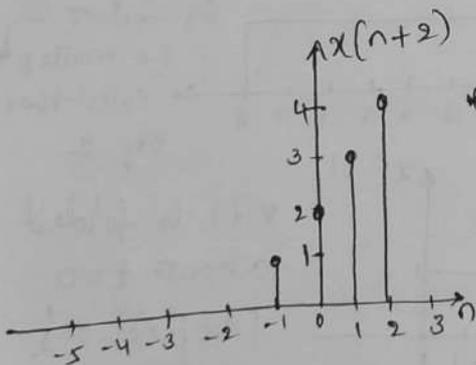
* $x[n]$ is translated & shifted to right by 2 units



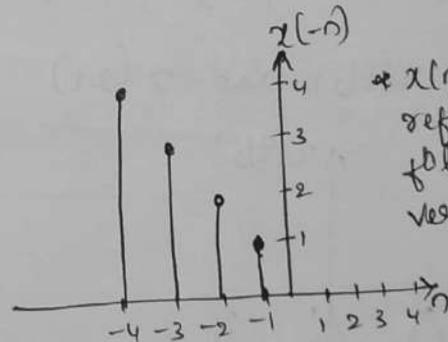
* $x[n]$ is scaled by factor 2 i.e. compressed by 2 units



* $x[n]$ is expanded by factor 4.



* $x[n]$ is shifted to left by two units



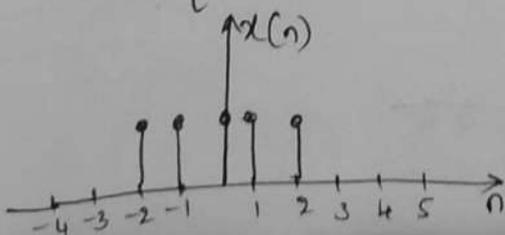
* $x[n]$ is reflected or folded about vertical axis

17) consider the D.T.S $x[n]$ defined by

$$x[n] = \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & \text{else where} \end{cases}$$

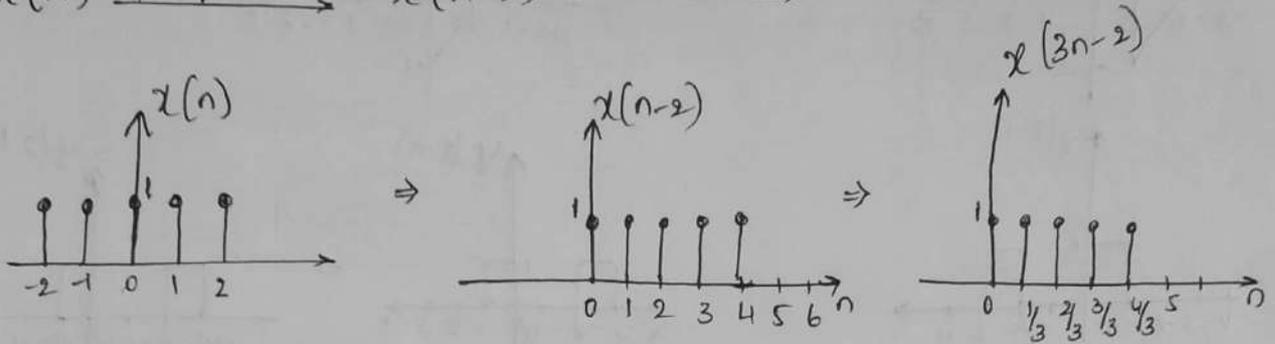
then find $y[n] = x[3n-2]$

soln:

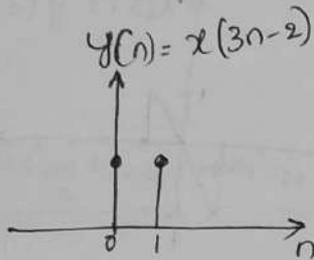


To find $y[n] = x[3n-2]$, two steps to be followed.

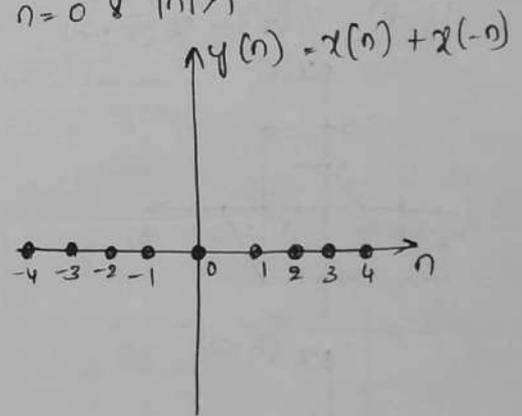
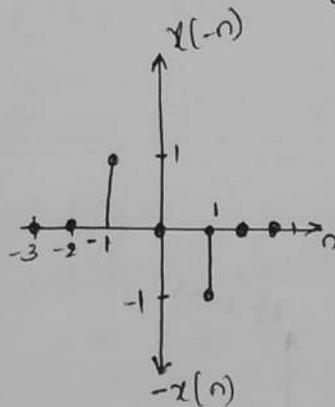
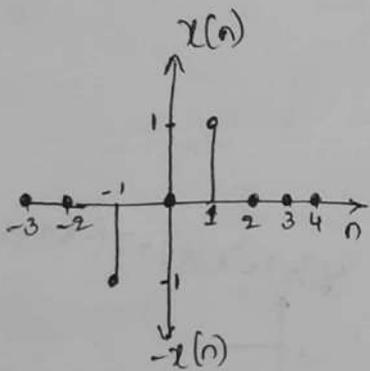
$$x[n] \xrightarrow{\text{shift 2 units}} x[n-2] \xrightarrow{\text{compress 3 units}} x[3n-2]$$



Since $y[n]$ is a discrete it can take only integer values so,



18) A D.T.S is defined by, $x[n] = \begin{cases} 1, & n=1 \\ -1, & n=-1 \\ 0, & n=0 \text{ \& } |n| > 1 \end{cases}$ find $y[n] = x[n] + x[-n]$

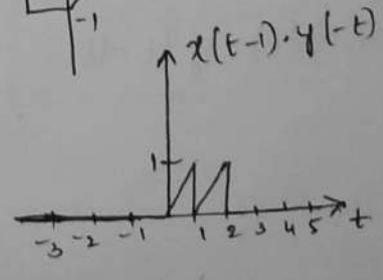
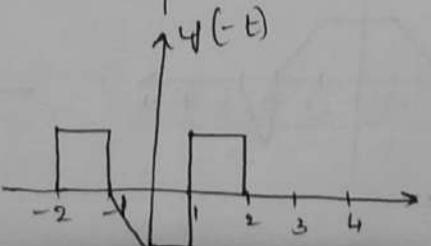
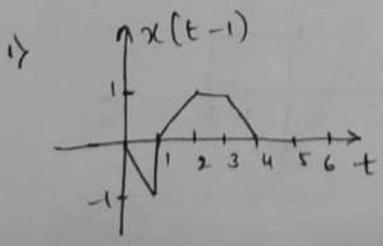
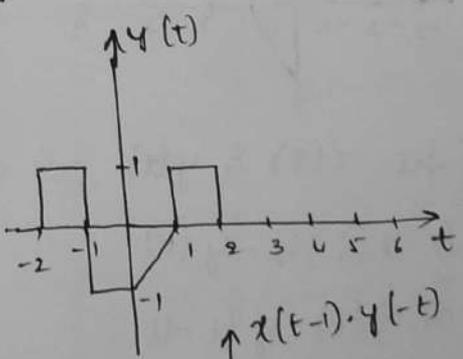
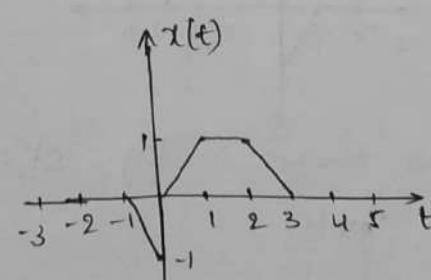


19) Let $x(t)$ and $y(t)$ is as shown in fig. sketch the following signals.

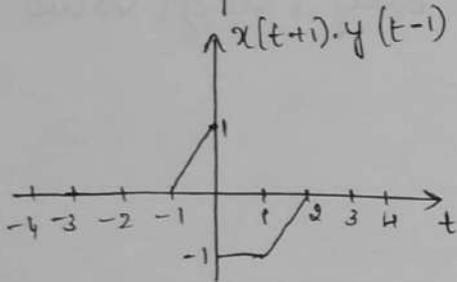
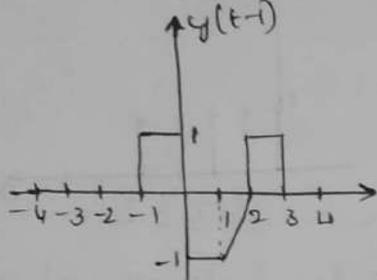
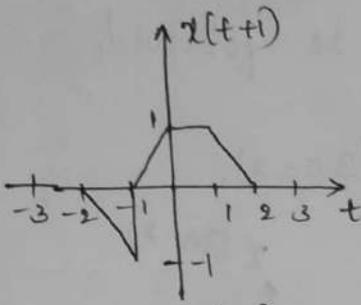
1) $x(t-1) \cdot y(t)$

2) $x(t+1) \cdot y(t-1)$

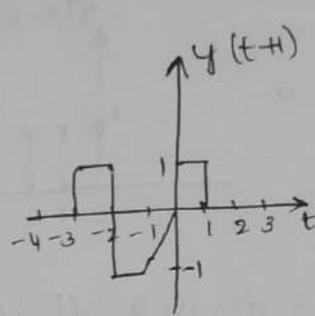
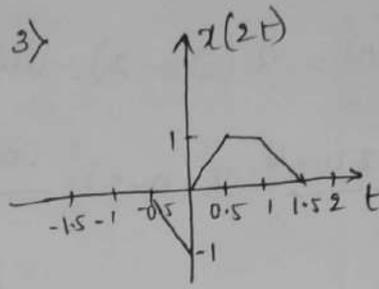
3) $x(2t) \cdot y(2t+1)$



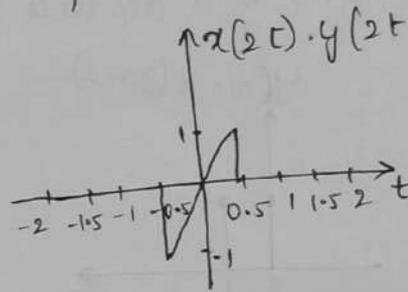
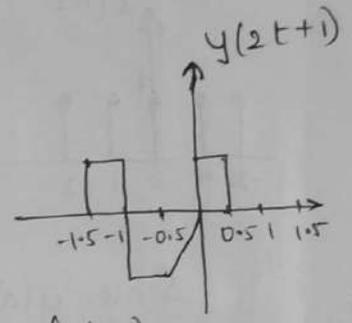
2)



3)

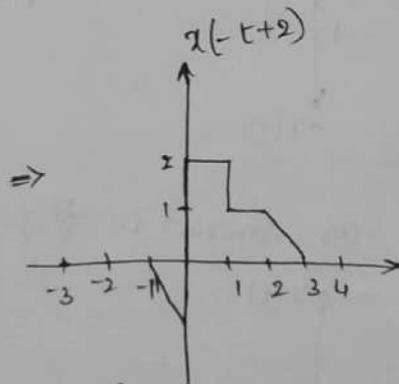
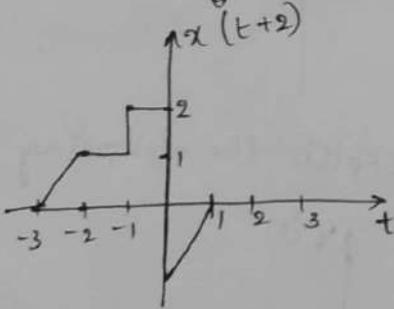
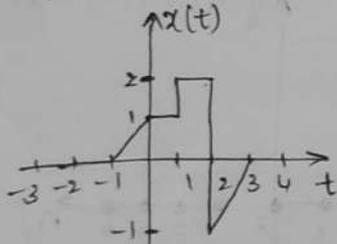


=>

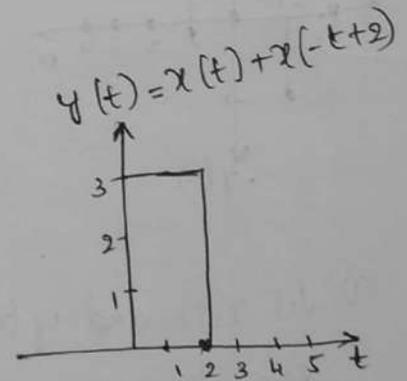


Q1) consider C.T.S $x(t)$ as shown in fig. Draw the signal

$$y(t) = x(t) + x(t+2)$$



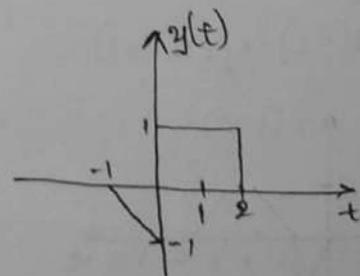
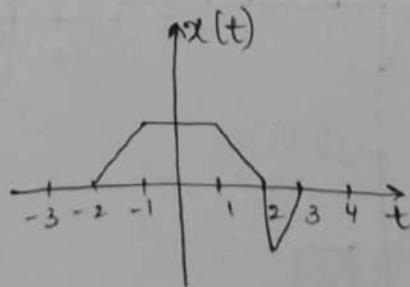
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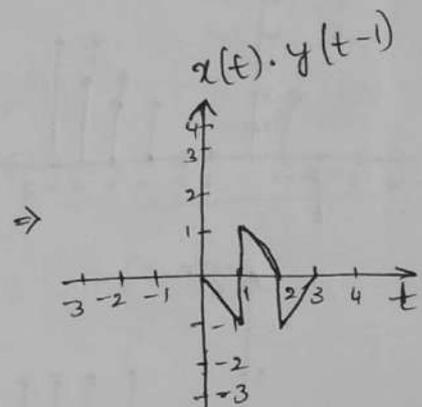
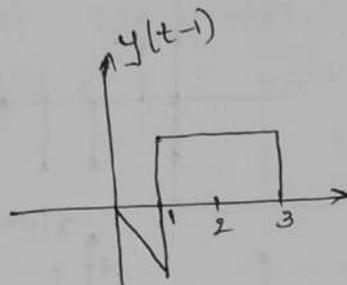
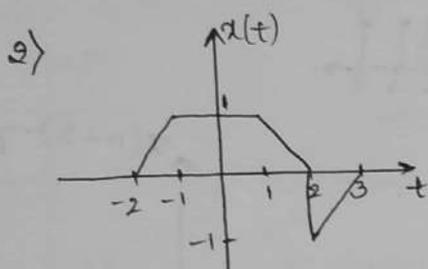
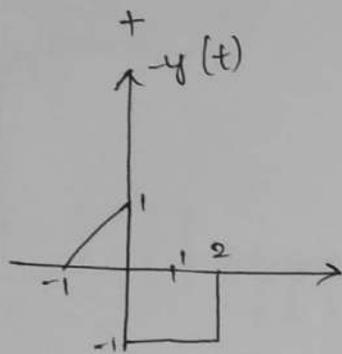
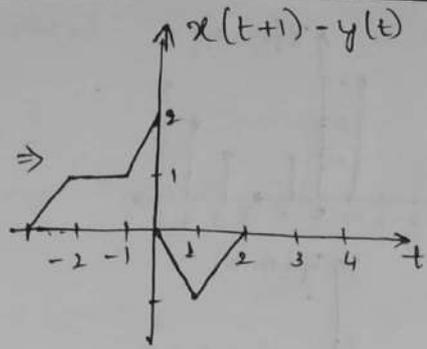
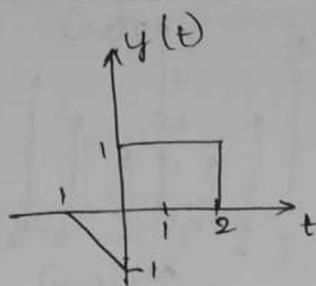
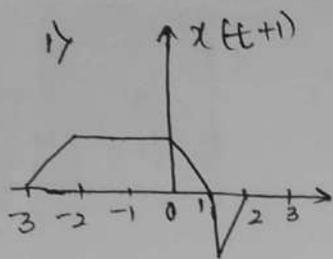


Q1) Let $x(t)$ & $y(t)$ is as shown in fig. sketch the signals.

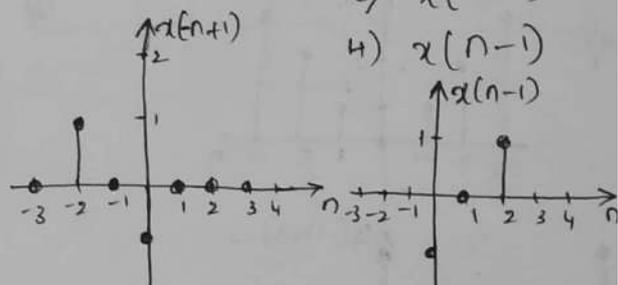
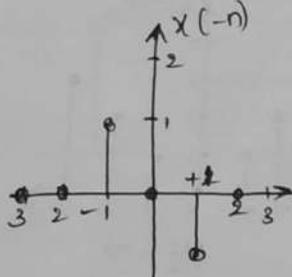
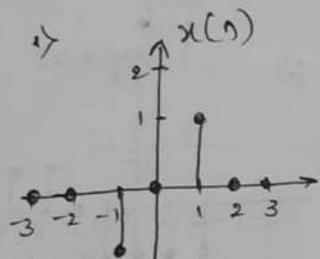
1) $x(t+1) - y(t)$

2) $x(t) \cdot y(t-1)$





3) A DTS is defined by $x(n) = \begin{cases} 1 & n=1 \\ -1 & \text{for } n=-1 \\ 0 & n=0 \text{ \& } |n| > 1 \end{cases}$ Sketch, if $x(n)$



2) $x(-n)$

3) $x(-n+1)$

4) $x(n-1)$

$x(n-1)$

4) Consider the signals $x(n]$ & $y[n)$ as shown in fig and sketch

1) $x(n-2) + y(n-2)$

2) $x(2n) + y(n-4)$

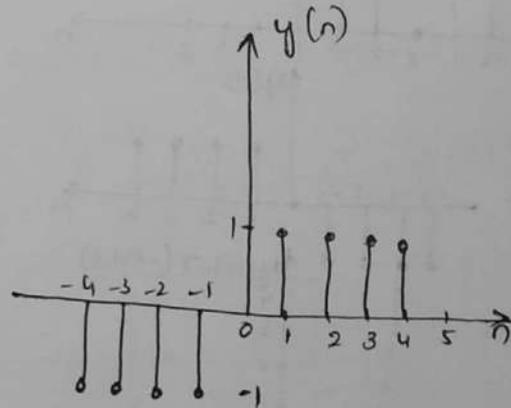
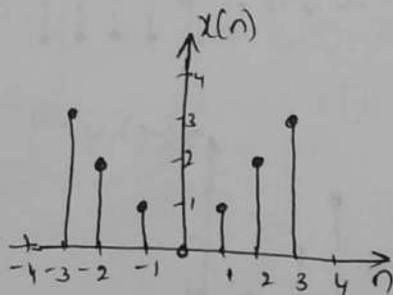
3) $x(n+2) \cdot y(n-2)$

4) $x(3-n) \cdot y[n)$

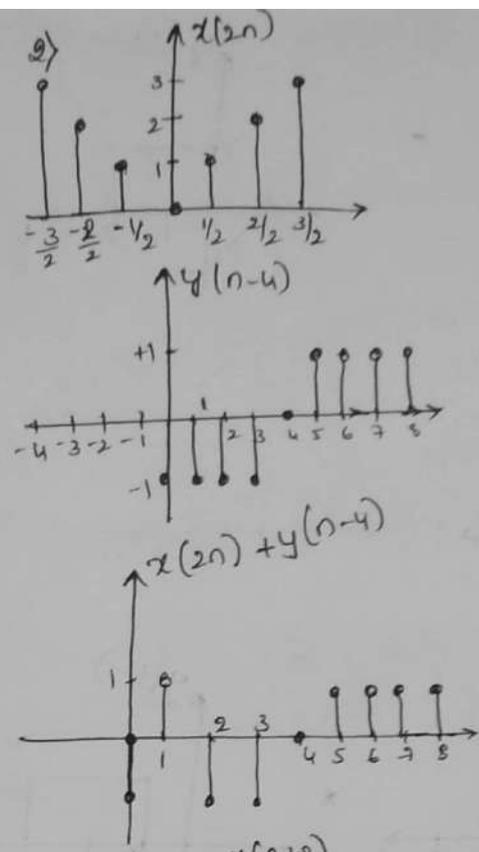
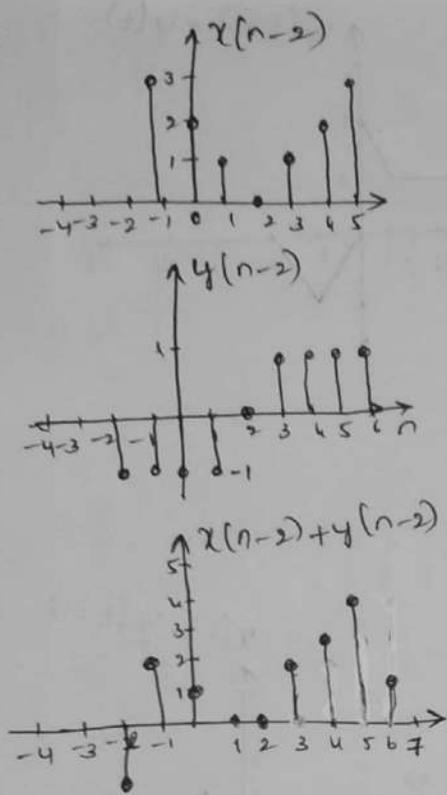
5) $x[-n] \cdot y[-n]$

6) $x[n] \cdot y[-2-n]$

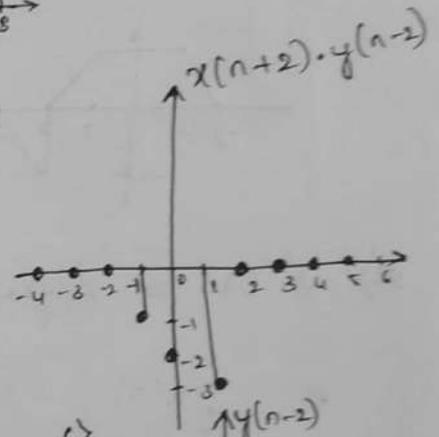
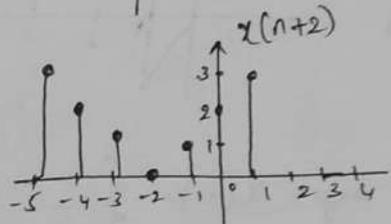
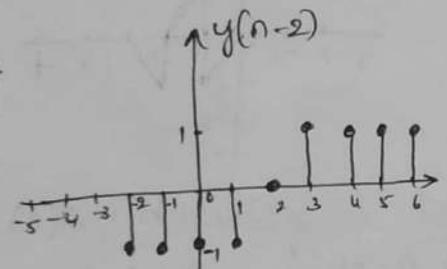
7) $x[n+2] \cdot y[6-n]$



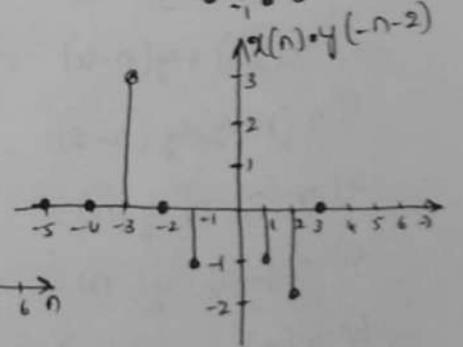
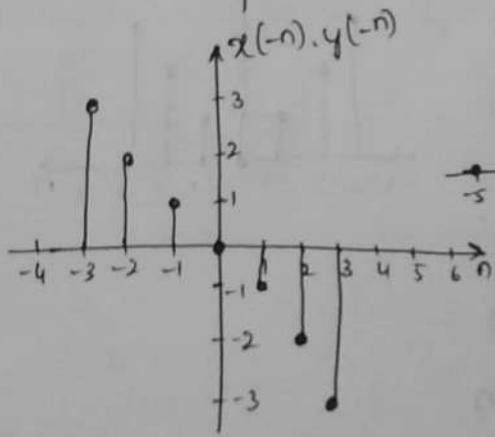
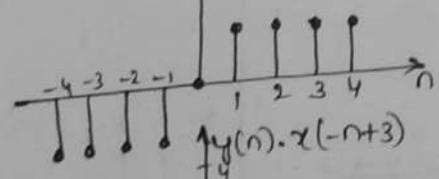
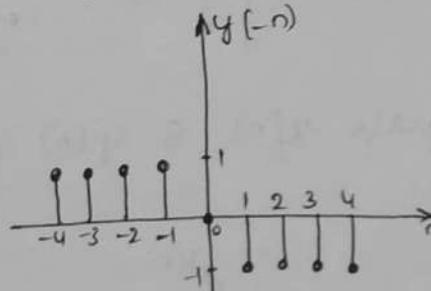
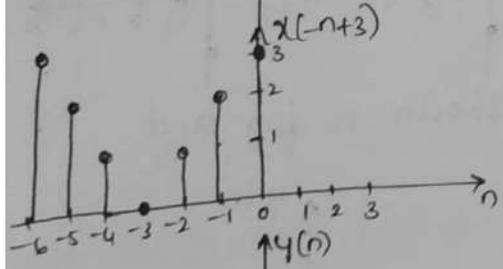
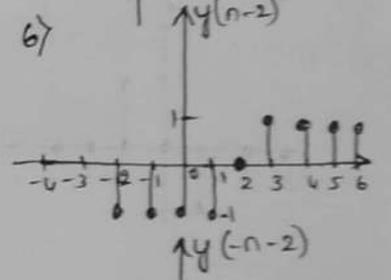
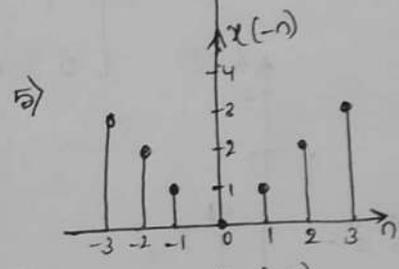
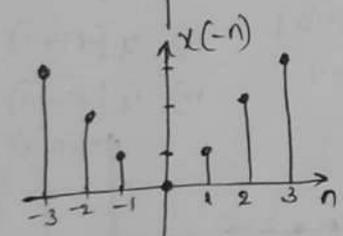
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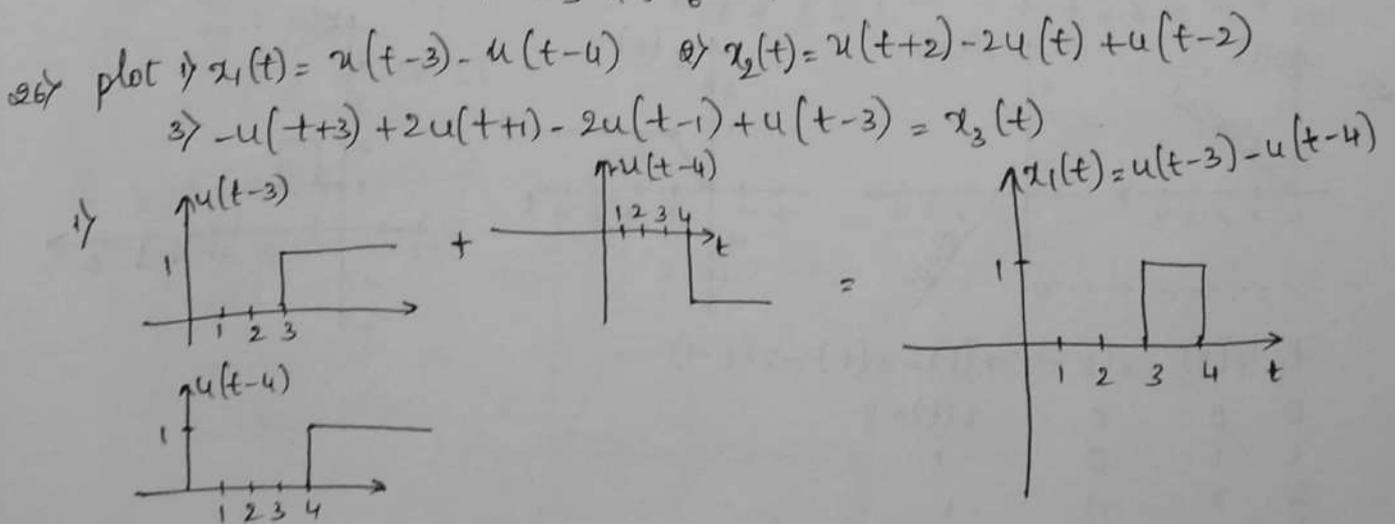
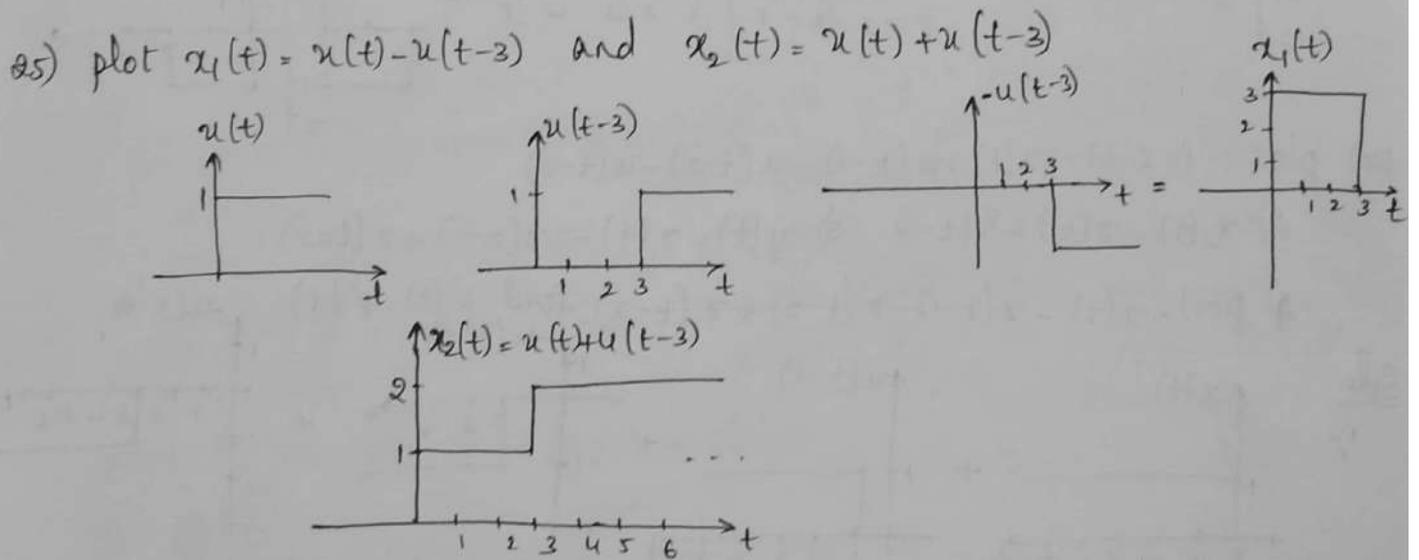
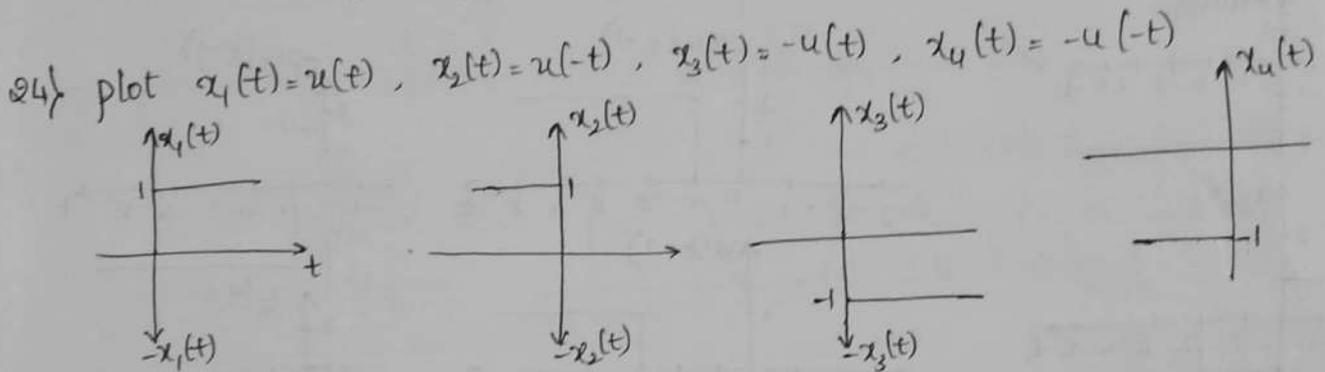
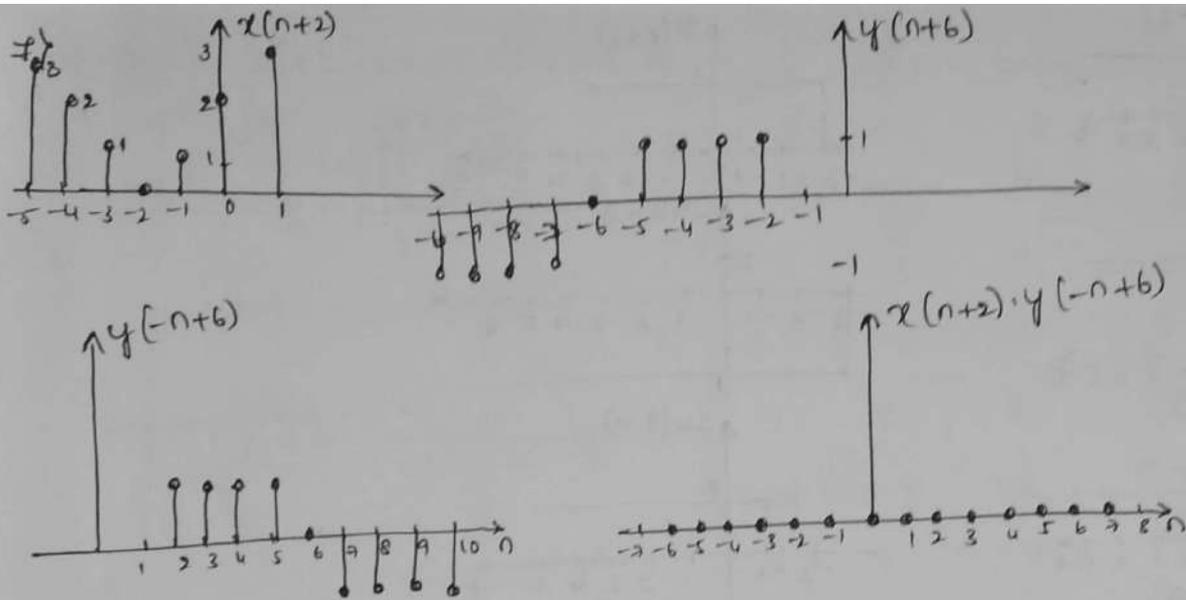


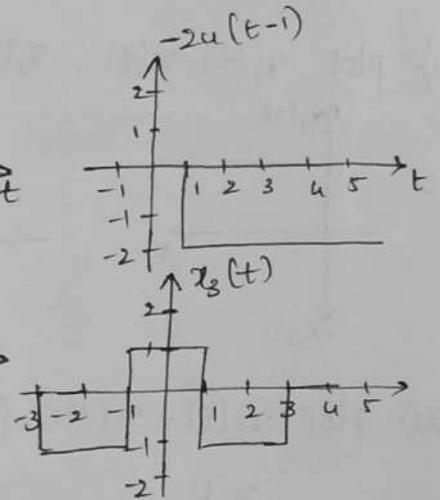
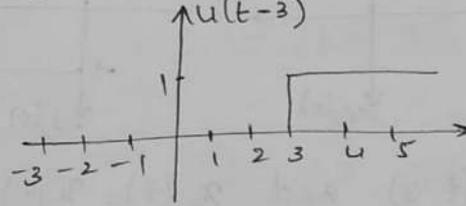
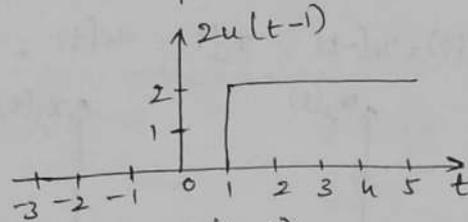
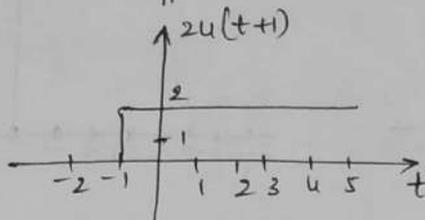
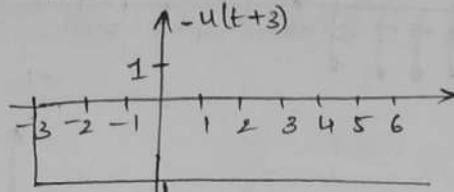
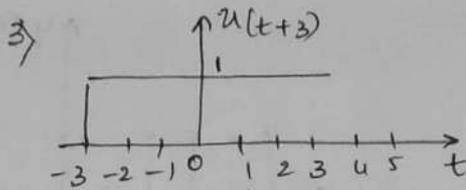
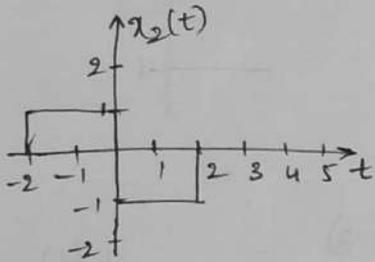
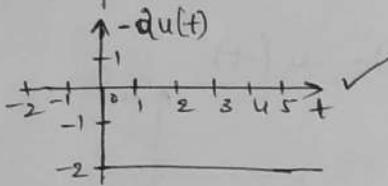
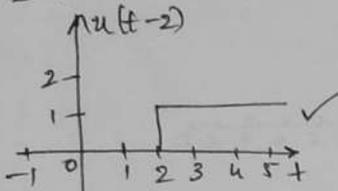
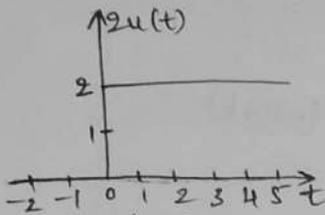
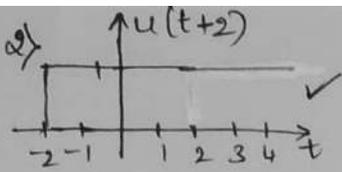
3)



4)



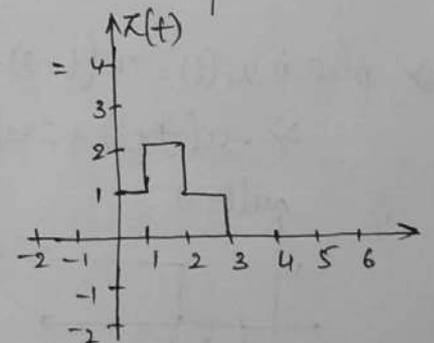
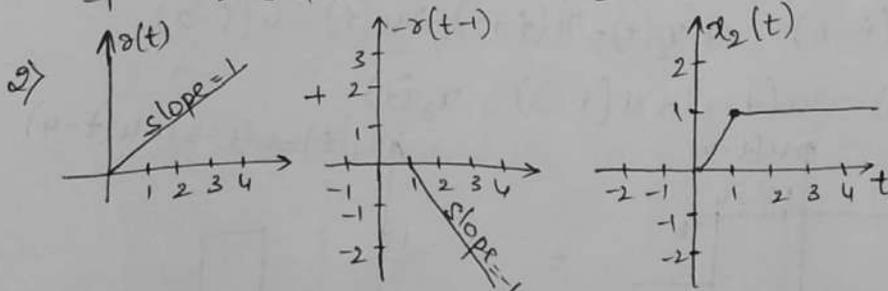
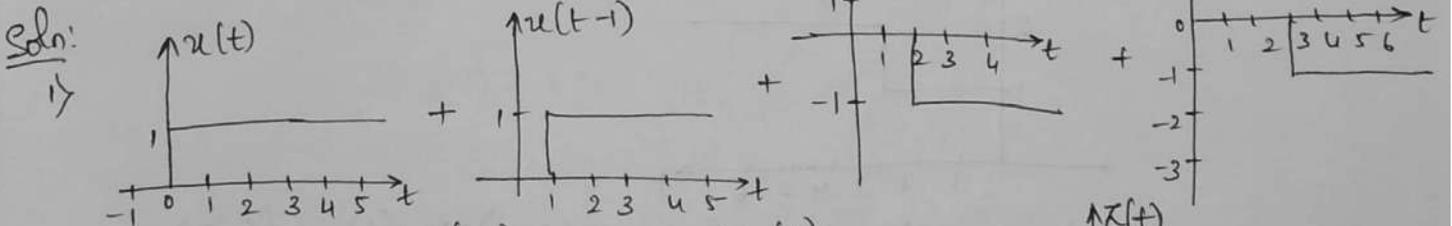




2f) plot: $x(t) = u(t) + u(t-1) - u(t-2) - u(t-3)$

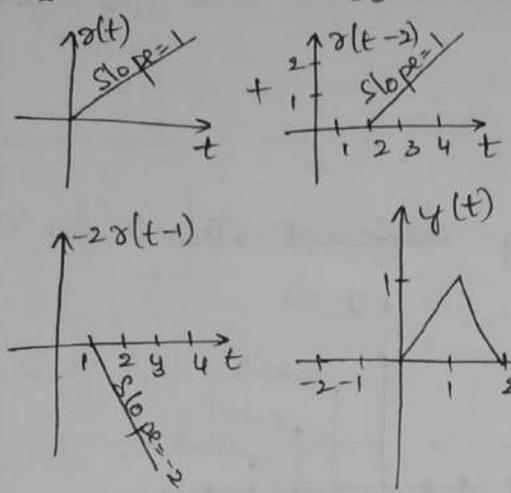
2) $x_2(t) = \sigma(t) - \sigma(t-1)$ 3) $y(t) = \sigma(t) - 2\sigma(t-1) + \sigma(t-2)$

4) $p(t) = \sigma(t) - \sigma(t-1) - \sigma(t-2) + \sigma(t-3)$, find $p'(t)$, $p''(t)$



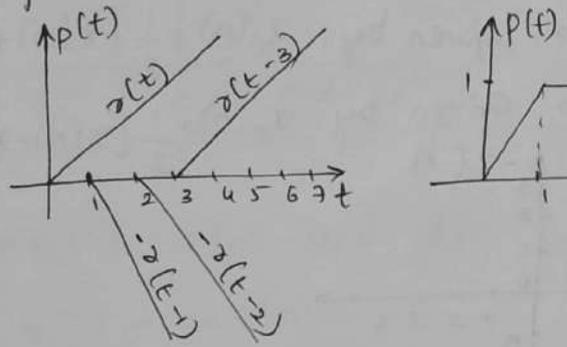
t	$\sigma(t)$	$-\sigma(t-1)$	$x(t) = \sigma(t) - \sigma(t-1)$
0	0	0	$x(t) = 0$
1	1	0	1
2	2	-1	1
4	4	-3	1
5	5	-5	1

3) $y(t) = r(t) - 2r(t-1) + r(t-2)$



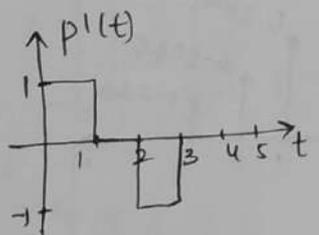
t	r(t)	-2r(t-1)	r(t-2)	y(t)
0	0	0	0	0
0.5	0.5	0	0	0.5
1	1	0	0	1
1.5	1.5	-1	0	0.5
2	2	-2	0	0
2.5	2.5	-3	0.5	0
3	3	-4	1	0
...

4) $p(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$

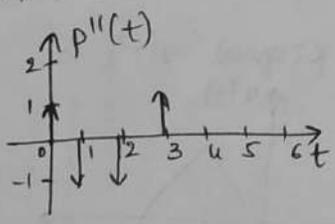


t	r(t)	-r(t-1)	-r(t-2)	r(t-3)	p(t)
0	0	0	0	0	0
0.5	0.5	0	0	0	0.5
1	1	0	0	0	1
1.5	1.5	-0.5	0	0	1
2	2	-1	0	0	1
2.5	2.5	-1.5	-0.5	0	0.5
3	3	-2	-1	0	0
3.5	3.5	-2.5	-1.5	0.5	0
4	4	-3	-2	1	0

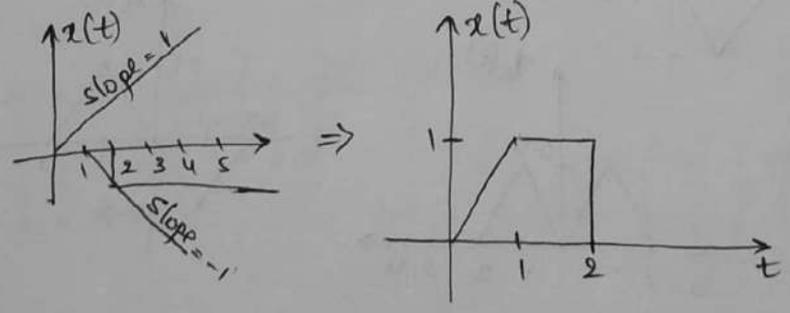
$p'(t) = u(t) - u(t-1) - u(t-2) + u(t-3)$



$p''(t) = \delta(t) - \delta(t-1) - \delta(t-2) + \delta(t-3)$



5) $x(t) = r(t) - r(t-1) - u(t-2)$

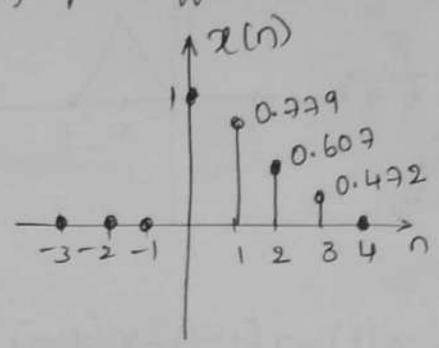


Q8) Find and sketch the even and odd components of the following signals.

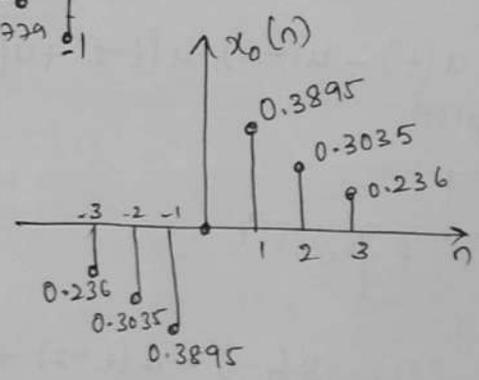
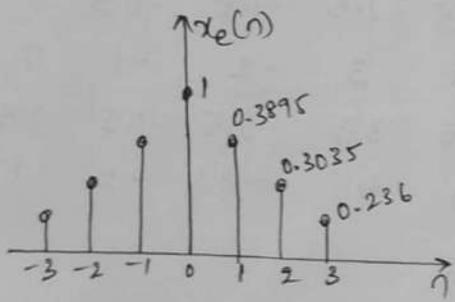
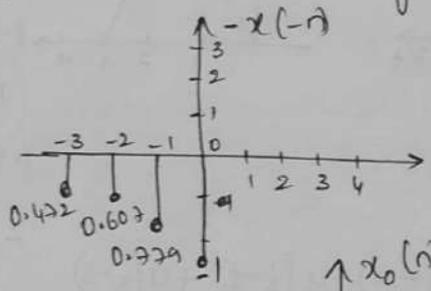
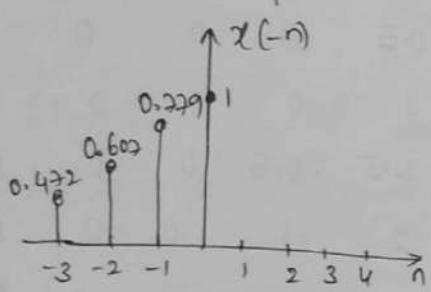
$x(n) = e^{-n/4} u(n), 0 \leq n \leq 3$

The signal is sketched by taking different values for 'n'

i.e $x(0) = e^{-0} x(n) = 1$
 $x(1) = e^{-1/4} x(n) = 0.779$
 $x(2) = e^{-1/2} x(n) = 0.607$
 $x(3) = e^{-3/4} x(n) = 0.472$

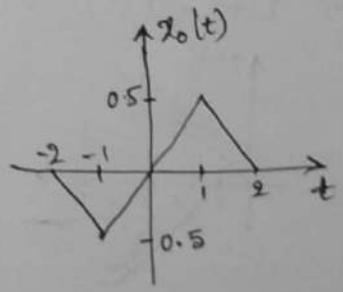
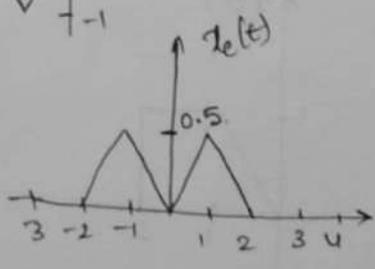
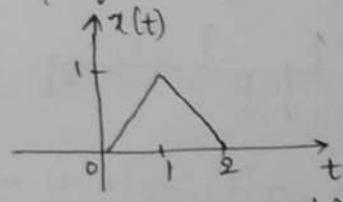
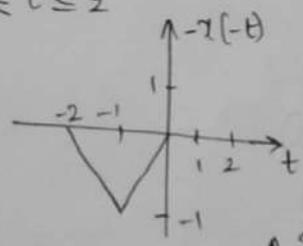
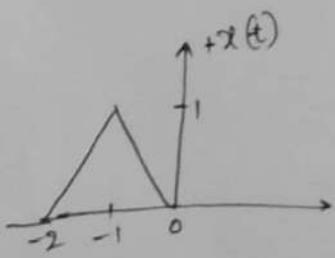


The even component of $x(n)$ is given by, $x_e(n) = \frac{1}{2} [x(n) + x(-n)]$
 odd component of $x(n)$ is given by $x_o(n) = \frac{1}{2} [x(n) - x(-n)]$



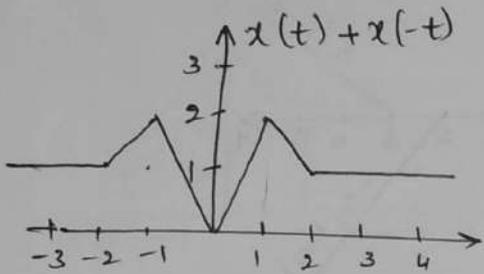
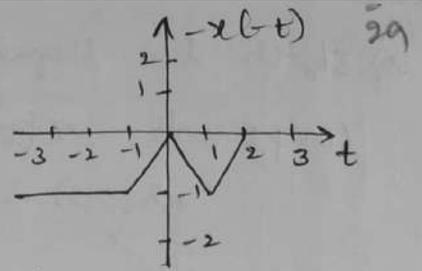
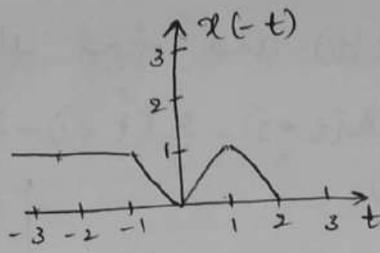
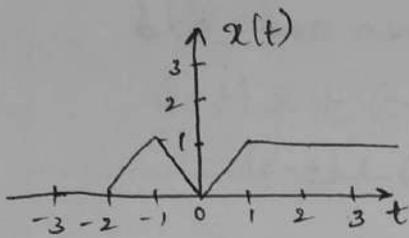
2) $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \end{cases}$

The given signal $x(t)$ is

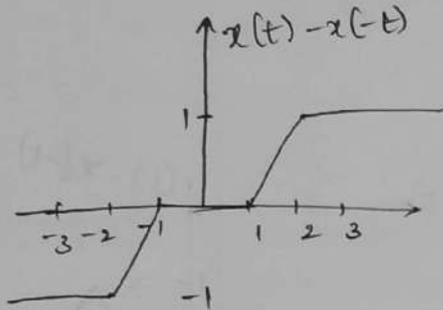
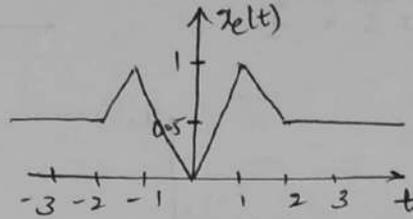


$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$
 $x_o(t) = \frac{1}{2} [x(t) - x(-t)]$

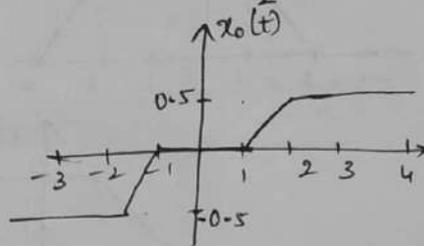
3)



$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

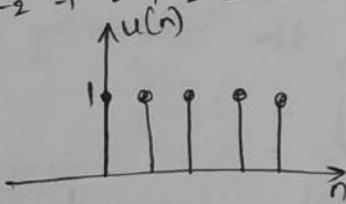
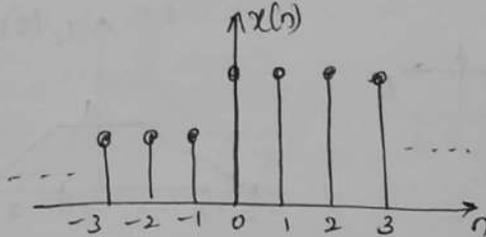
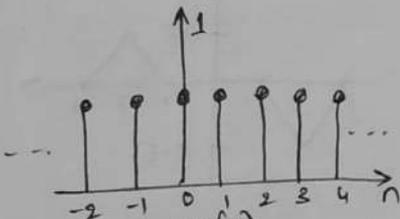


$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



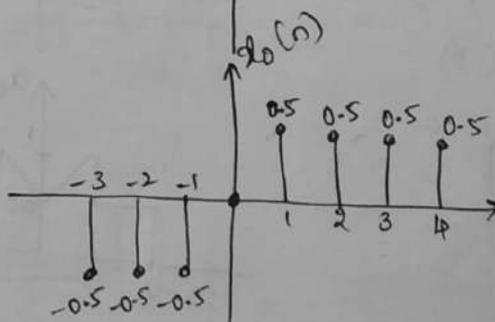
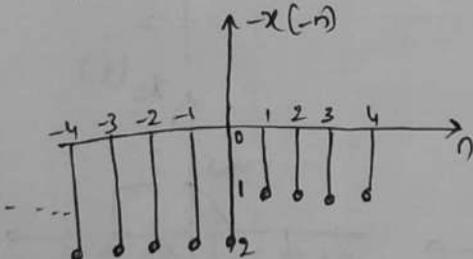
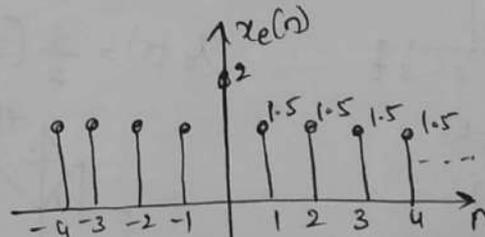
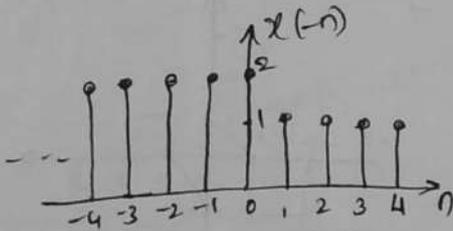
4) sketch $x(n)$ and plot even and odd components of

$$x(n) = 1 + u(n)$$

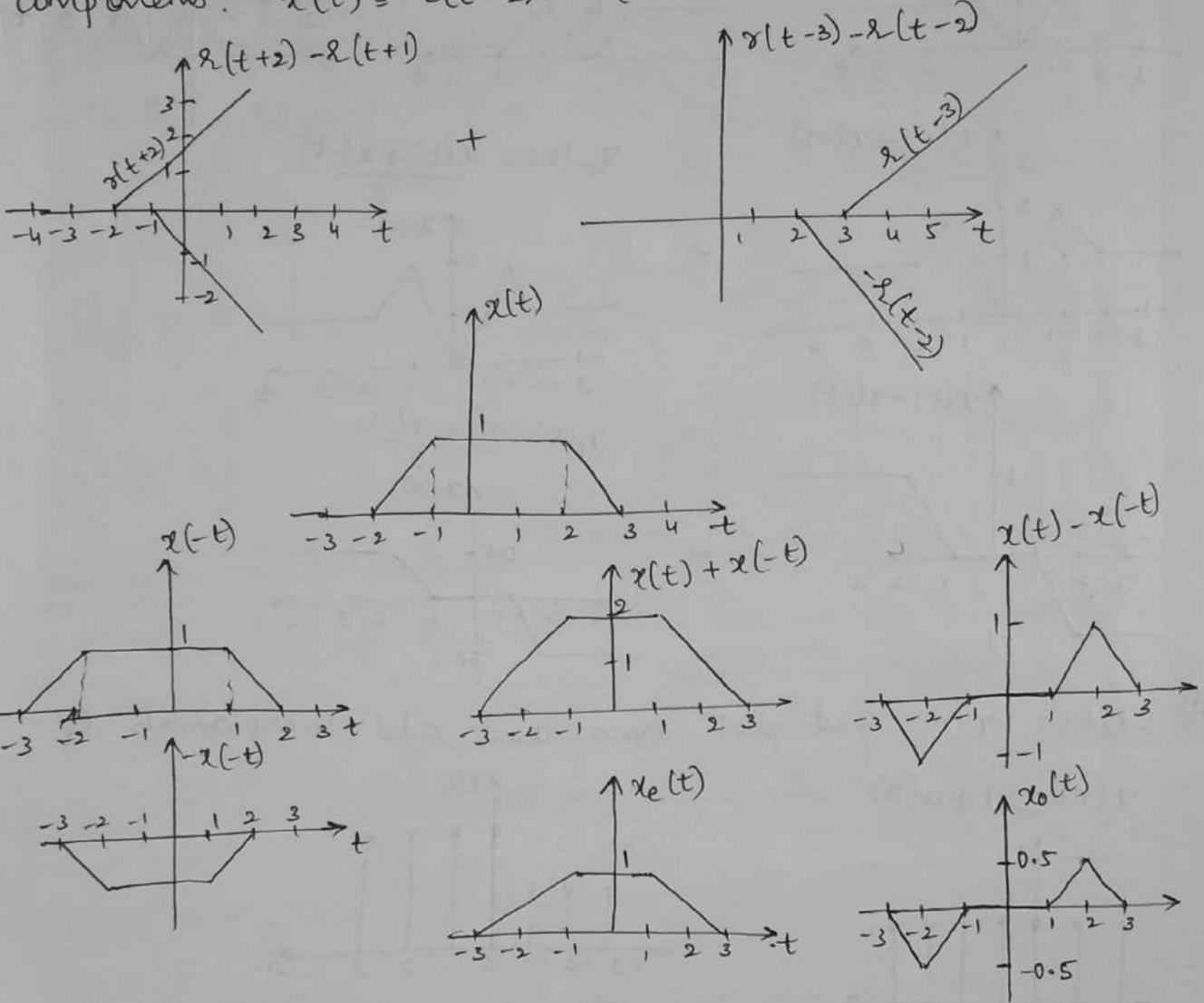


$$x_e(n) = \frac{1}{2} [x(n) + x(-n)]$$

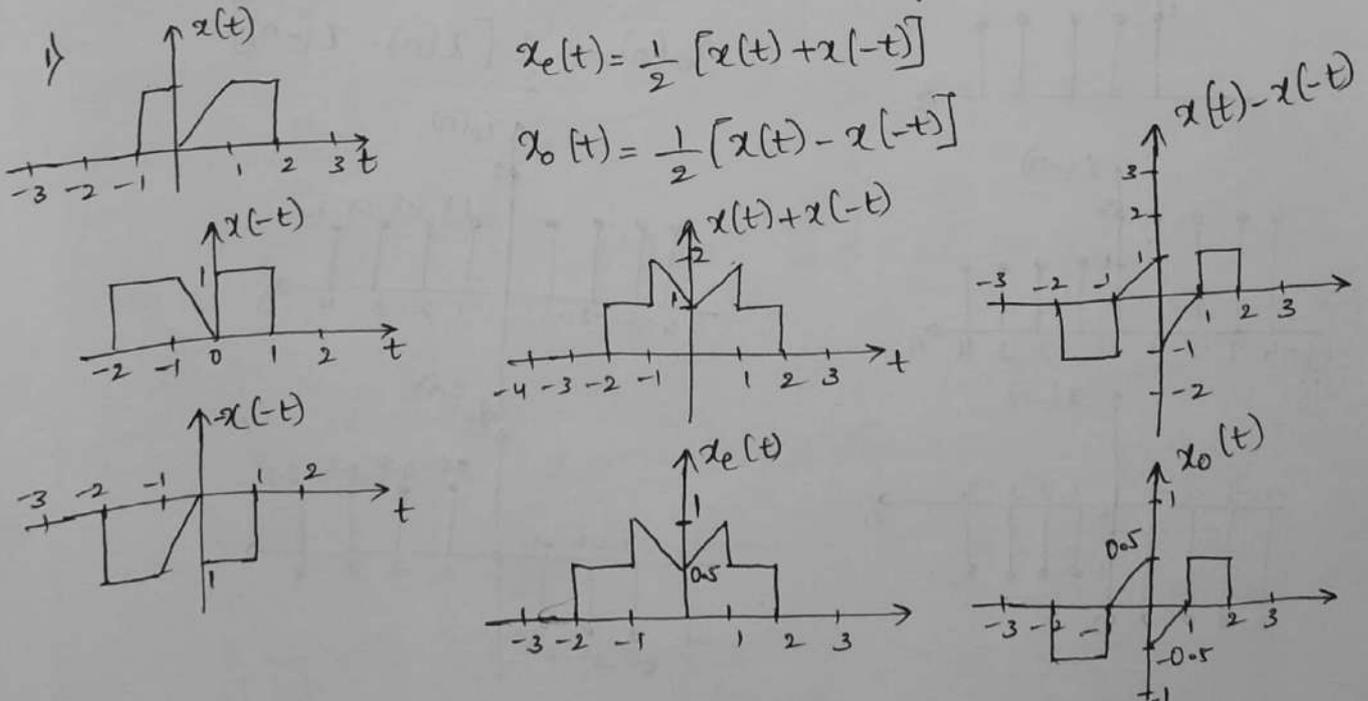
$$x_o(n) = \frac{1}{2} [x(n) - x(-n)]$$



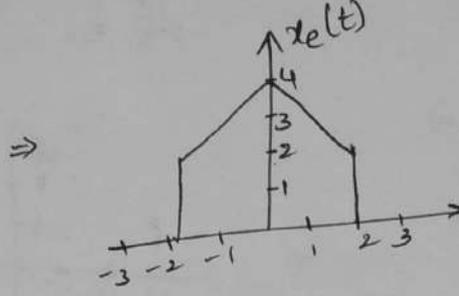
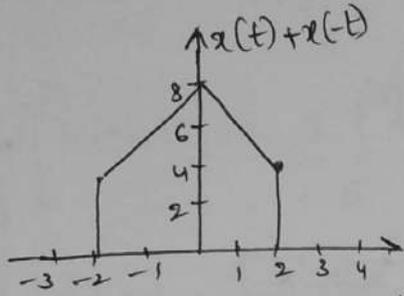
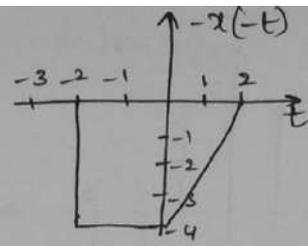
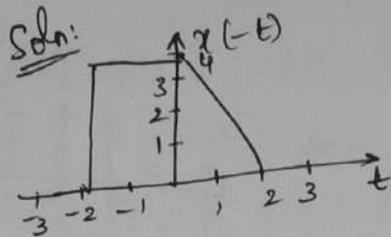
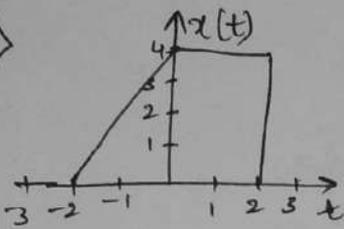
30) Sketch the signal $x(t)$ and find its even and odd components. $x(t) = x(t+2) - x(t+1) - x(t-2) + x(t-3)$



31) Sketch even and odd components of the following

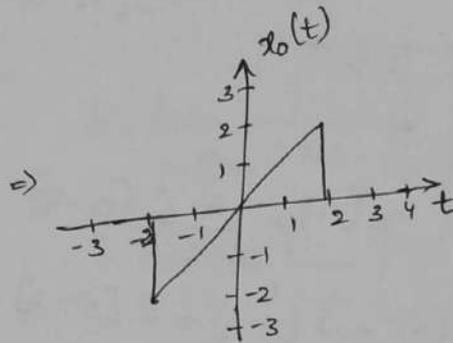
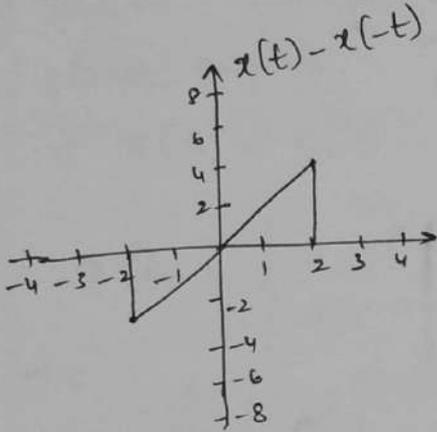


2)

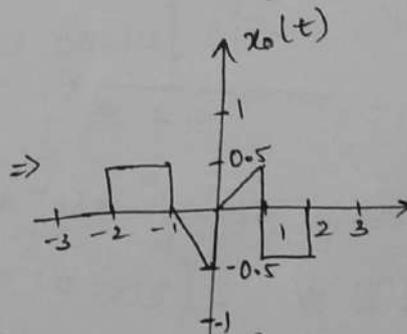
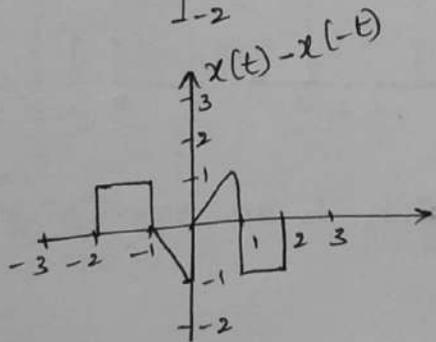
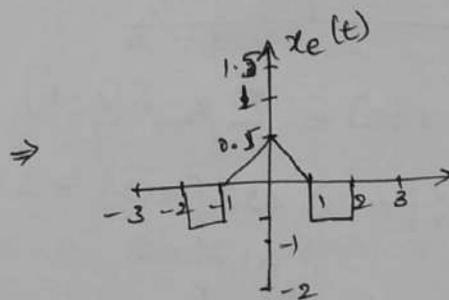
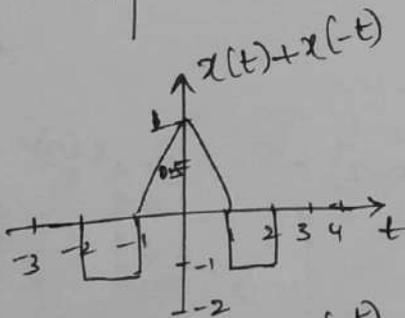
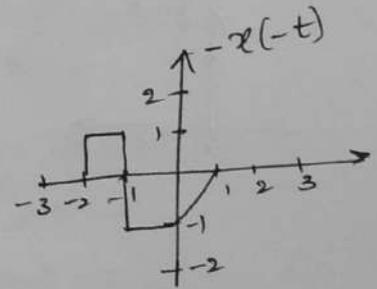
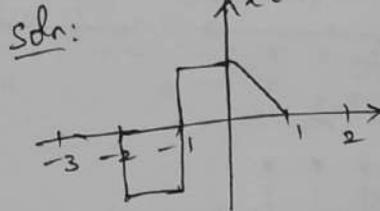
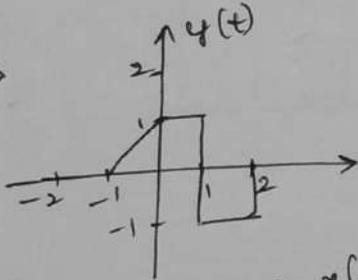


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

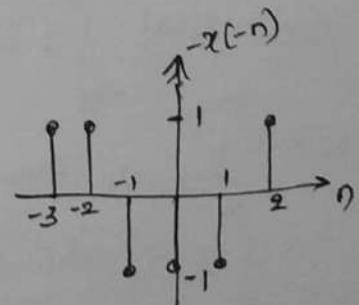
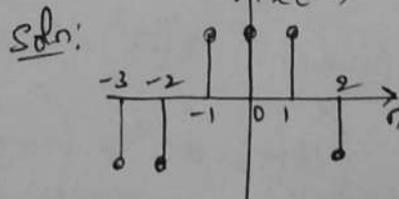
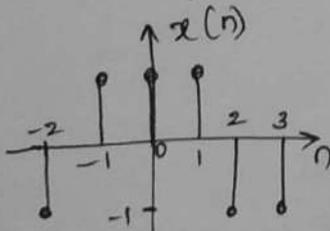
$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

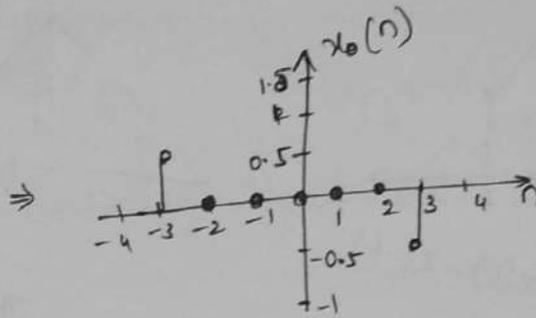
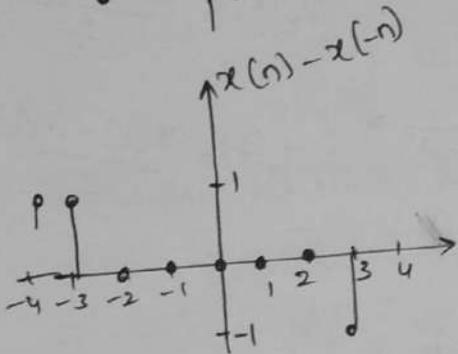
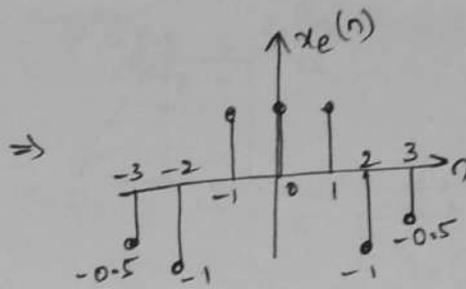
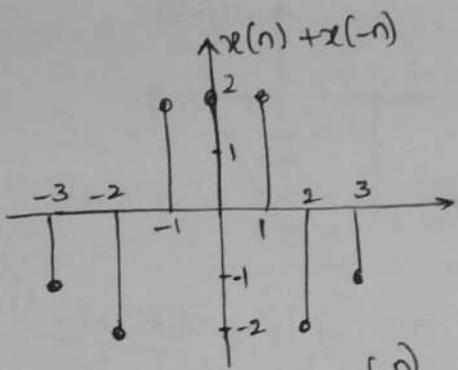


3)



4)

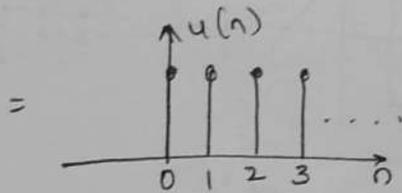
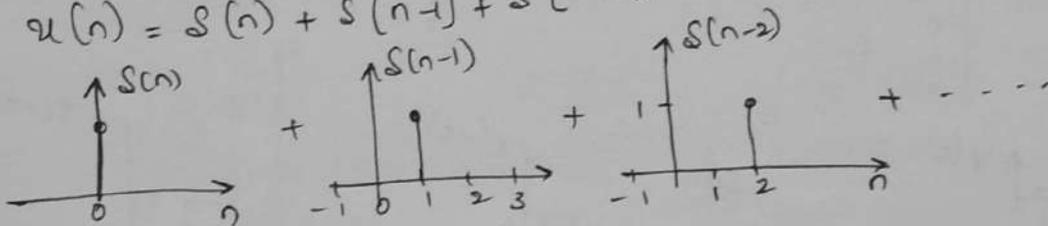




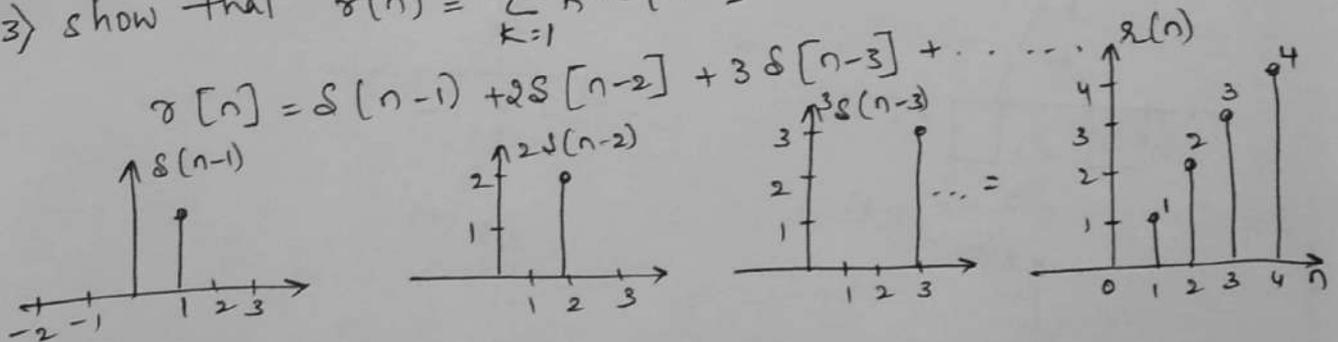
32) prove that $u(n) = \sum_{k=0}^{\infty} \delta[n-k]$

Soln:

$$u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots$$



33) show that $r(n) = \sum_{k=1}^{\infty} k \cdot \delta[n-k]$



34) For any arbitrary signal $x(t)$ which is an even signal, show that $\int_{-\infty}^{\infty} x(t) \cdot dt = 2 \int_0^{\infty} x(t) \cdot dt$

Soln:

w.k.T for an even signal, $x(-t) = x(t)$

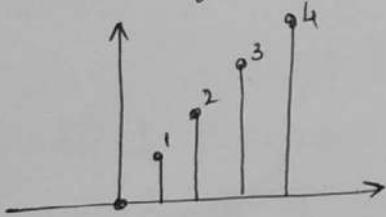
$$\int_{-\infty}^{\infty} x(t) \cdot dt = \int_{-\infty}^0 x(t) \cdot dt + \int_0^{\infty} x(t) \cdot dt$$

$$\begin{aligned}
 &= \int_{-\infty}^0 x(-t) \cdot dt + \int_0^{\infty} x(t) \cdot dt \\
 &= \int_0^{\infty} x(t) \cdot dt + \int_0^{\infty} x(t) \cdot dt \\
 \int_{-\infty}^{\infty} x(t) \cdot dt &= 2 \int_0^{\infty} x(t) \cdot dt.
 \end{aligned}$$

35) Determine whether the following signals are energy & power signals.

1) $x(n) = x[n]$

$$x[n] = \begin{cases} 0, & n < 0 \\ n, & n \geq 0 \end{cases}$$



$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left[\sum_{n=-N}^0 0 + \sum_{n=0}^N n^2 \right]$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N n^2$$

$$= \lim_{N \rightarrow \infty} \frac{N(N+1)(2N+1)}{6(2N+1)}$$

$$P = \lim_{N \rightarrow \infty} \frac{N(N+1)}{6} = \infty$$

$$\begin{aligned}
 E &= \sum_{n=-\infty}^{\infty} |x(n)|^2 \\
 &= \sum_{n=-\infty}^0 0 + \sum_{n=0}^{\infty} n^2
 \end{aligned}$$

$$\boxed{P = \infty}$$

so, Ramp signal $x(n)$, is neither a power signal nor a energy signal.

$$\boxed{E = \infty}$$

36) $y(n) = A$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N A^2$$

$$= \lim_{N \rightarrow \infty} \frac{A^2}{2N+1} \sum_{n=-N}^N 1$$

$$= \lim_{N \rightarrow \infty} \frac{A^2}{2N+1} \times 2N+1$$

$$\boxed{P = A^2}$$

$$E = \sum_{n=-\infty}^{\infty} A^2$$

$$= A^2 \sum_{n=-\infty}^{\infty} 1$$

$$= A^2 \times \infty$$

$$\boxed{E = \infty}$$

Systems Viewed as an Interconnection of Operations. ①

A system may be viewed as an interconnection of operations that transforms an i/p signal into an o/p signal with properties different from those of the i/p signal.

The signal may be continuous & discrete time & mixture of both. The overall action of the s/m is denoted by operator H & T

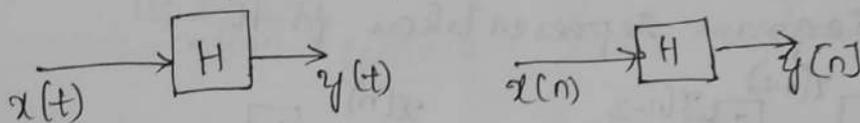
Application of signal $x(t)$ as an i/p to the system yields o/p signal $y(t)$ as

$$y(t) = H\{x(t)\}$$

Application of signal $x(n)$ as an i/p to the discrete time system yields o/p signal $y(n)$ as

$$y[n] = H\{x(n)\}$$

Block Diagram representation is



There are two types of block diagram representation for operator H & systems.

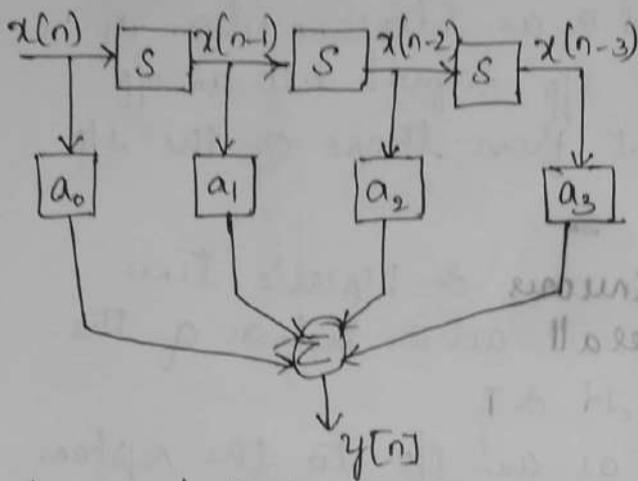
1) Using Cascade implementation

2) Using cascade implementation & parallel implementation.

ex: The o/p of the Discrete time s/m is related to i/p $x(n)$ as, $y[n] = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) + a_3 x(n-3) \rightarrow \text{①}$

Let S^k represents the s/m that shifts the i/p $x(n)$ by 'k' time & units to produce $x(n-k)$. The Block diagram representation for the s/m shown in eq-① is

1) Using cascade implementation

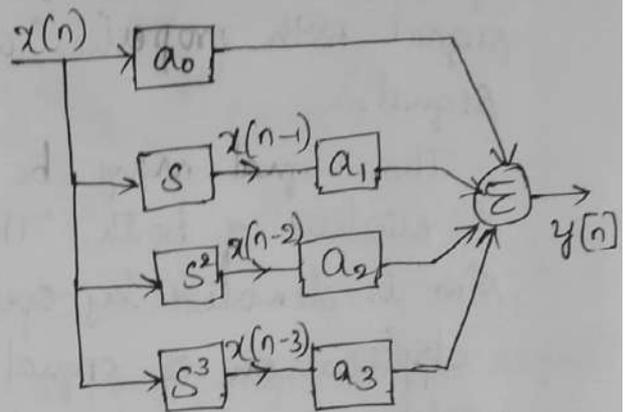


* Uses identical time shifters

Overall operation or operator H

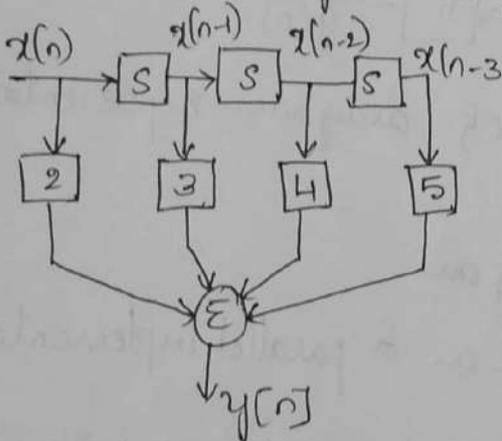
$$H = a_0 + a_1s + a_2s^2 + a_3s^3$$

2) Using cascade or parallel implementation.



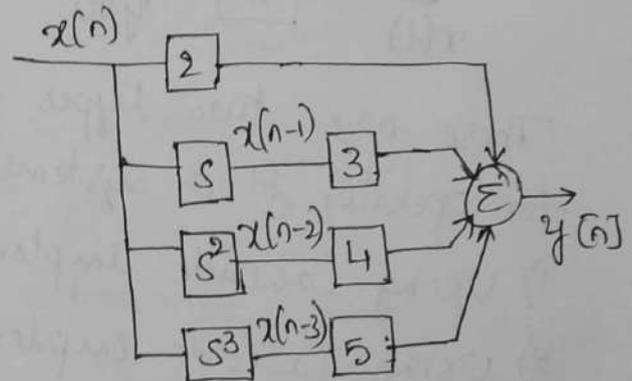
* Uses different time shifters namely $s, s^2, \& s^3$

Q. Discrete time systems is represented by the following i/p-o/p relation $y[n] = 2x[n] + 3x[n-1] + 4x[n-2] + 5x[n-3]$. Develop the block diagram representation for H .



Cascade Implementation

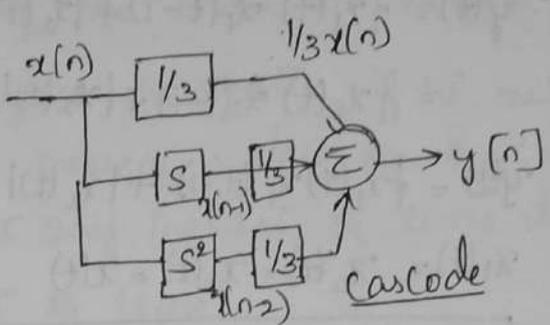
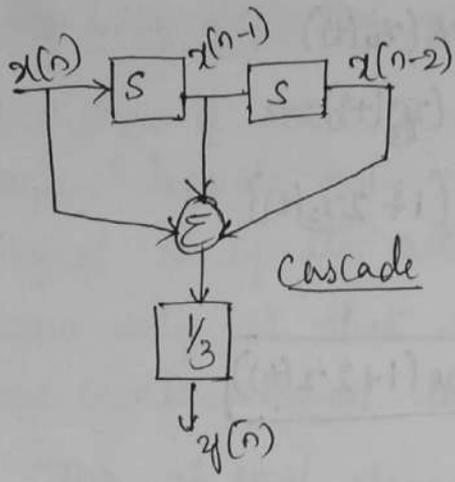
$$H = 2 + 3s + 4s^2 + 5s^3$$



Cascade implementation

Q. Find the overall operator of a system whose output signal $y[n]$ is given by $y[n] = \frac{1}{3} [x[n] + x[n-1] + x[n-2]]$ also draw the block diagram representation.

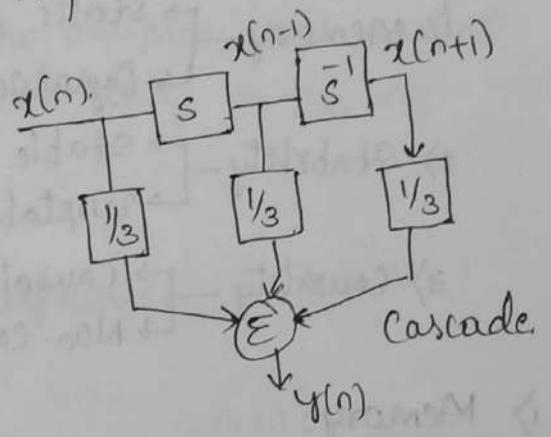
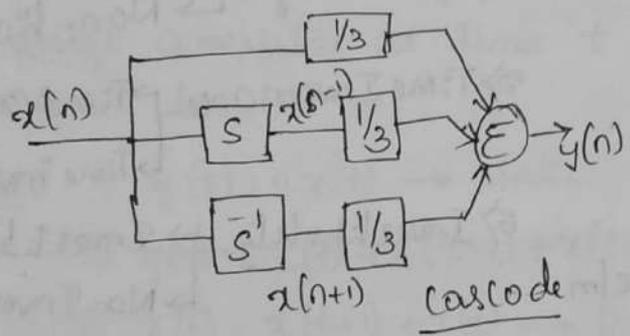
$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$



$$H = \frac{1}{3} (1 + S + S^2)$$

$$H = \frac{1}{3} + \frac{1}{3} S + \frac{1}{3} S^2$$

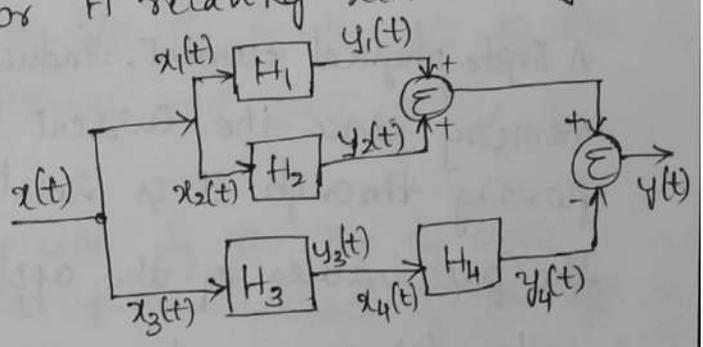
Q. Find the overall operator of a system whose output signal $y(n)$ is given by $y(n) = \frac{1}{3} [x(n+1) + x(n) + x(n-1)]$. Also draw the block diagram representation.



$$H = \frac{1}{3} S^{-1} + \frac{1}{3} + \frac{1}{3} S$$

$$H = \frac{1}{3} [S^{-1} + 1 + S]$$

Q. A system consists of several subsystems connected as shown in fig. Find the operator H relating $x(t)$ and $y(t)$ for the subsystem operators



$$H_1: y_1(t) = x_1(t) x_1(t-1)$$

$$H_2: y_2(t) = |x_2(t)|$$

$$H_3: y_3(t) = 1 + 2x_3(t)$$

$$H_4: y_4(t) = \cos[x_4(t)]$$

soln: From the fig, $y(t) = \{y_1(t) + y_2(t)\} - y_4(t)$

$$\begin{aligned} \therefore y(t) &= \{x_1(t)x_1(t-1) + |x_2(t)|\} - \cos(x_4(t)) \\ &= \{x_1(t)x_1(t-1) + |x_2(t)|\} - \cos(y_3(t)) \\ y(t) &= \{x_1(t)x_1(t-1) + |x_2(t)|\} - \cos(1 + 2x_3(t)) \\ x_1(t) &= x_2(t) = x_3(t) = x(t) \\ \therefore H: y(t) &= x(t)x(t-1) + |x(t)| - \cos(1 + 2x(t)) \end{aligned}$$

Properties of Systems

The properties of s/m describes the characteristics of the operator 'H' representing the s/m. The different properties of system are,

1) Memory $\begin{cases} \rightarrow \text{Static s/m} \\ \rightarrow \text{Dynamic s/m} \end{cases}$

2) Stability $\begin{cases} \rightarrow \text{stable s/m} \\ \rightarrow \text{unstable s/m} \end{cases}$

3) Causality $\begin{cases} \rightarrow \text{Causal s/m} \\ \rightarrow \text{Non-Causal s/m} \end{cases}$

4) Linearity $\begin{cases} \rightarrow \text{Linear s/m} \\ \rightarrow \text{Non-linear s/m} \end{cases}$

5) Time Invariance $\begin{cases} \rightarrow \text{Time Variant} \\ \rightarrow \text{Time invariant} \end{cases}$

6) Invertibility $\begin{cases} \rightarrow \text{Invertible s/m} \\ \rightarrow \text{Non-Invertible s/m} \end{cases}$

1) Memory:

A System is said to possess memory if its o/p signal depends on past and/or future values of the input signal. then the s/m is also referred as the Dynamic system

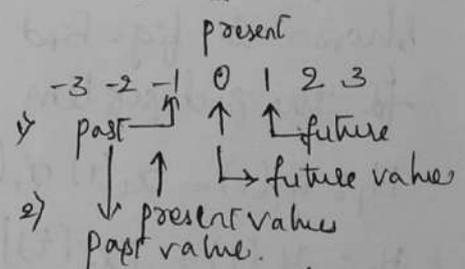
ex: sequential logic circuits, Flipflops, counters etc

A basic physical element, inductor has a memory hence the current $i(t)$ flowing through it is related to the past values of the applied v/g.

v/g $v(t)$ is given by

$$i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

where L is the inductance of the inductor. The current through a inductor at time t depends on all past values



the vlg $v(t)$. the memory of the inductor extend upto ∞ .³

A system is said to memoryless or static if its output signal depends only on the present value of the input signal & if its output at any instant of time depends upon only at that instant of time.

ex: combinational logic ckt. AND, NOT, OR etc

The physical element is Resistor, which doesn't possess any memory since the current $i(t)$ flowing through it is in response to the applied vlg is defined by.

$$i(t) = \frac{1}{R} v(t).$$

where R is the resistance of the resistor. the current through a resistor at time t depends only on the vlg at that time t .

ex: $\hookrightarrow y(t) = 2x(t) \rightarrow$ static

$\Rightarrow y(t) = x(t) + x(t-1) + x(t-2) \rightarrow$ Dynamic

$\Rightarrow y(t) = x(t+1) + x(t) \rightarrow$ Dynamic

$\hookrightarrow y(t) = e^{-(t+1)} \cdot x(t) \rightarrow$ static because $e^{-(t+1)}$ is a co-efficient an i/p.

\Rightarrow Causality:

A system is said to be causal if the o/p of the s/m is independent of future values of i/p. or if the o/p of the s/m is dependent only on the present and past values of the i/p.

A system is said to be non-causal if the o/p at any instant of time depends on the future values of the i/p signal.

ex: 1) $y(t) = x(t) \rightarrow$ Causal

2) $y(t) = x(t) + x(t-1) \rightarrow$ Causal

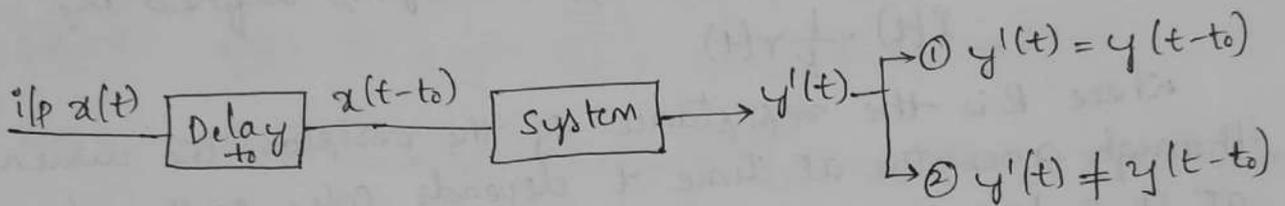
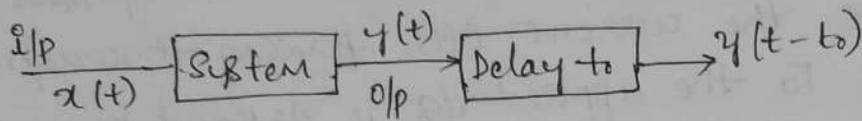
3) $y(n) = \frac{1}{3} [x(n+1) + x(n) + x(n-1)] \rightarrow$ Non-causal.
 \hookrightarrow future value

Anticausal s/m: If the output of the s/m depends only on the future values of i/p. This is exactly opposite to the causal s/m

ex: $y(t) = x(t+2)$

3) Time Invariance

This property of systems is very important in LTI s/m's.

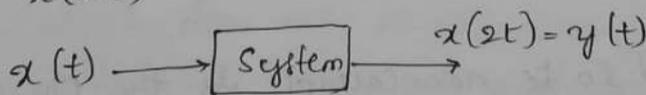


if $y'(t) = y(t-t_0) \Rightarrow$ Time invariant s/m

\Rightarrow if $y'(t) \neq y(t-t_0) \Rightarrow$ Time variant s/m.

The system is said to be time invariant if a time delay & advance of the i/p leads to an identical time shift in the output signal. This implies that the time invariant systems responds identically no matter when the input signal is applied or the characteristic of a time invariant s/m do not change with time.

ex: $y(t) = x(2t)$



To check whether it is time variant or time invariant

Step-1) $y(t) \xrightarrow{t_0} y(t-t_0) = x[2(t-t_0)]$

$= x[2t - 2t_0] \rightarrow \textcircled{1}$

Step-2) $x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{\text{System}} x[2(t-t_0)]$
 $y(t) = x[2t - 2t_0] \rightarrow \textcircled{2}$

Comparing ① & ②

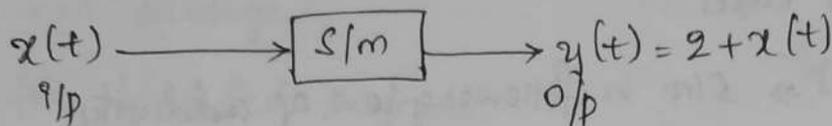
$$y(t) = y(t-t_0)$$

∴ SLM is time variant.

Note: Always time scaling \rightarrow Time variant

Amplitude scaling \rightarrow Time Invariant

e) $y(t) = 2 + x(t)$



To check whether time variant or Time invariant

Step 1: $y(t) \xrightarrow{t_0} y(t-t_0) = 2 + x(t-t_0) \rightarrow \text{①}$

Step 2: $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{S/LM} \rightarrow y(t) = 2 + x(t-t_0) \rightarrow \text{②}$

∴ By ① & ② $y(t) = y(t-t_0)$

∴ The given SLM is time invariant.

4) Linearity:

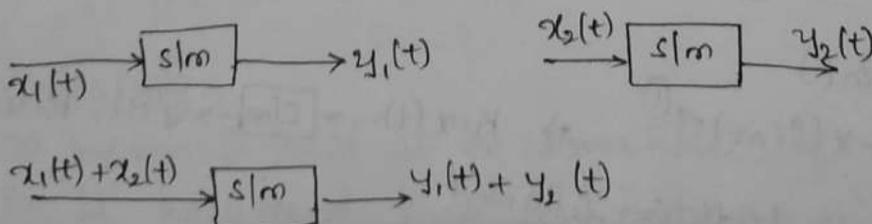
A SLM is said to be linear in terms of the SLM i/p (excitation) $x(t)$ and the SLM o/p (response) $y(t)$ if it satisfies the following property, i.e. Law of Superposition, and Law of Homogeneity

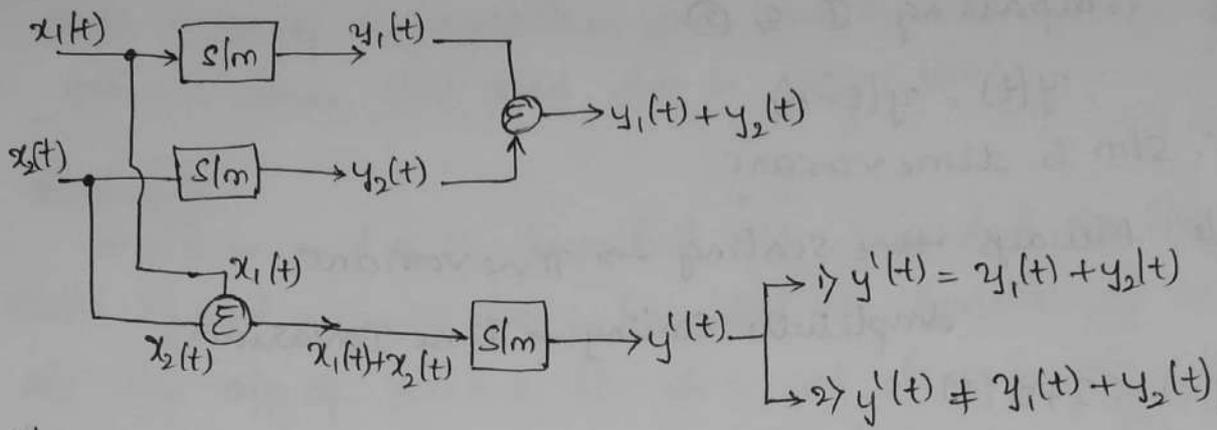
→ Law of Superposition is also called as Law of Additivity [LOA]

→ Law of Homogeneity is also called as Law of multiplication of [LOH] Scalars multiplication.

A SLM which does not satisfy any of the above properties then it is called as non-linear systems.

Law of Superposition [LOA]



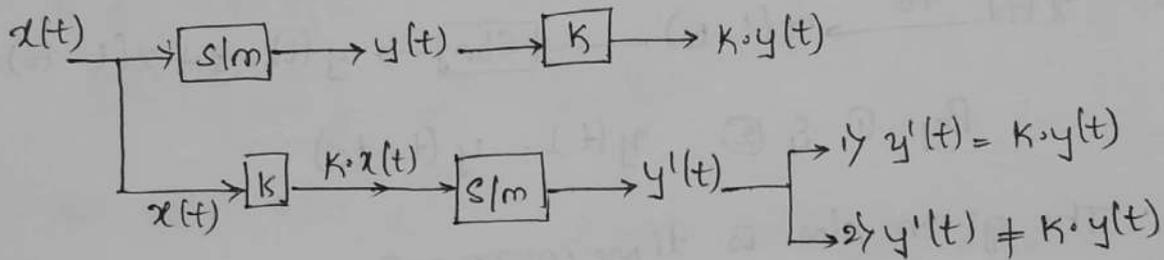


s/m is same in all cases.

if $y'(t) = y_1(t) + y_2(t) \Rightarrow$ s/m is following law of additivity

$y'(t) \neq y_1(t) + y_2(t) \Rightarrow$ s/m is not following law of additivity.

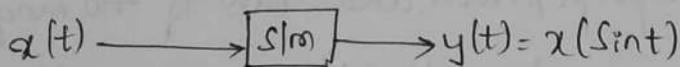
Law of Homogeneity [LOH]



if $y'(t) = K \cdot y(t) \Rightarrow$ s/m is following Law of Homogeneity

$y'(t) \neq K \cdot y(t) \Rightarrow$ s/m is not following Law of Homogeneity

ex: $y(t) = x(\sin t)$



\Rightarrow LOA

$$y_1(t) = x_1(\sin t) \quad y_2(t) = x_2(\sin t)$$

$$y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t) \rightarrow \textcircled{1}$$

$$x_1(t) + x_2(t) \rightarrow \text{s/m} \rightarrow y'(t) = x_1(\sin t) + x_2(\sin t) \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2} \Rightarrow \text{LOA is followed.}$$

\Rightarrow LOH

$$y(t) = x(\sin t)$$

$$\Rightarrow K \cdot y(t) = K \cdot x(\sin t) \rightarrow \textcircled{1}$$

$$\Rightarrow K \cdot x(t) \rightarrow \text{s/m} \rightarrow y'(t) = K \cdot x(\sin t) \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2} \Rightarrow \text{LOH is followed}$$

Both Law of Superposition and Law of Homogeneity are followed hence the given s/m is linear s/m.

⇒ Stability

A s/m is said to be bounded i/p, bounded o/p [BIBO] stable if and only if every bounded i/p results in bounded o/p. The o/p of such a s/m does not diverge if the i/p does not diverge.

i.e for $\forall |x(t)| \leq M_x < \infty$ for all t

o/p $|y(t)| \leq M_y < \infty$ for all t

ex: for bounded signals.

dc value, $\sin t$, $\cos t$, $u(t)$

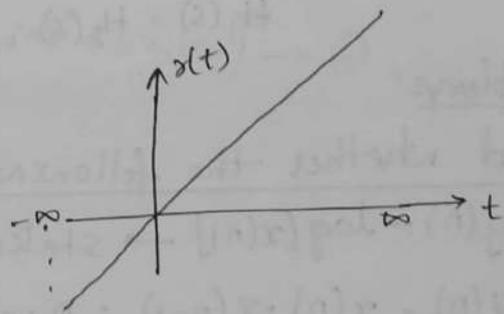
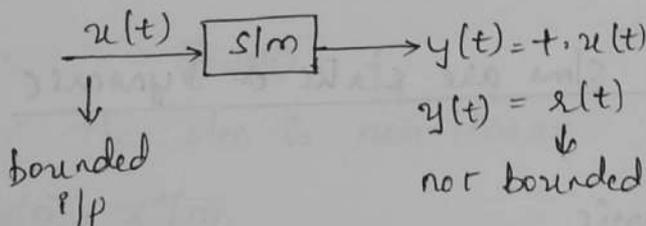
constant, -1 to 1 , 1 to -1 , 0 or 1

M_x & M_y represents finite positive numbers.

ex: if $y(t) = t \cdot x(t)$

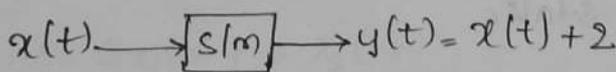


let $x(t) = u(t)$

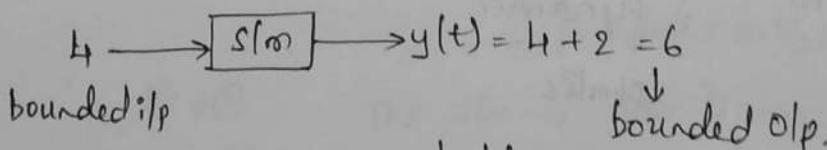


\therefore s/m is unstable

2) $y(t) = x(t) + 2$



$x(t) = 4$



\therefore The s/m is stable.

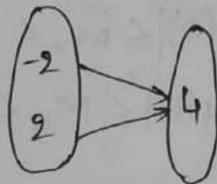
6) Invertibility:

A system is said to be invertible if the input of the s/m can be recovered from s/m output. Alternatively, a s/m

is said to be invertible if distinct ip's leads to distinct outputs. i.e for any invertible s/m there should be one to one mapping. b/w ip & op at each and every instant of time.

ex: $y(t) = x^2(t)$

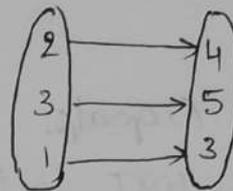
$x(t)$ $y(t)$



∴ It is not invertible.

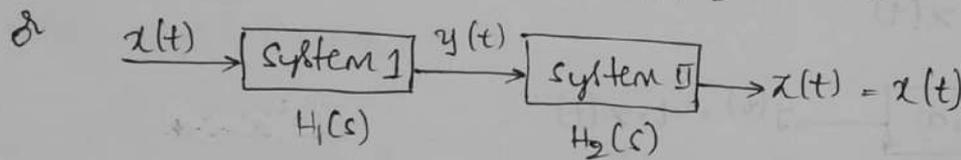
$y(t) = x(t) + 2$

$x(t)$ $y(t)$



It has one to one mapping.

∴ It is invertible.



$$H_1(s) \cdot H_2(s) = 1$$

Problems:

I) Find whether the following s/m's are static & Dynamic

- 1) $y(n) = \log[x(n)] \rightarrow$ static
- 2) $y(n) = x(n) \cdot x(n-1) :$ Dynamic
- 3) $y(n) = x^2(n) + x(n) :$ static
- 4) $y(n) = A \cdot x(n) :$ static
- 5) $y(n) = n \cdot x(n) + B \cdot x^3(n) :$ static
- 6) $y(n) = x(n) + 3x(n-1) :$ Dynamic
- 7) $y(n) = \sum_{k=0}^n x(n-k) :$ Dynamic
- 8) $y(n) = e^{x^2(n)} :$ static

II. Determine whether the following systems are linear & Non-linear

1) $y(n) = n \cdot x(n)$

$a x_1(n) \rightarrow y_1(n) = n a x_1(n)$

$b x_2(n) \rightarrow y_2(n) = n b x_2(n)$

$$y_1(n) + y_2(n) = n \cdot ax_1(n) + n \cdot bx_2(n) \\ = n \{ ax_1(n) + bx_2(n) \} \rightarrow \textcircled{1}$$

$$\underbrace{ax_1(n) + bx_2(n)}_{x(n)} \longrightarrow \boxed{\text{slm}} \longrightarrow y(n) = n \cdot \{ x(n) \} \\ = n \cdot \{ ax_1(n) + bx_2(n) \} \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

\therefore The slm is linear.

$$g) y(n) = x(n^2)$$

$$a \cdot x_1(n) \xrightarrow{H} y_1(n) = a \cdot x_1(n^2) \quad b \cdot x_2(n) \xrightarrow{H} y_2(n) = b \cdot x_2(n^2)$$

$$y_1(n) + y_2(n) = a x_1(n^2) + b x_2(n^2) \rightarrow \textcircled{1}$$

$$x(n) = a x_1(n) + b x_2(n)$$

$$x(n) \longrightarrow \boxed{\text{slm}} \longrightarrow y(n) = x(n^2) \\ = [a x_1(n) + b x_2(n)]^2 \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2}$$

\therefore The slm is non-linear

$$g) y(n) = x^2(n)$$

$$a \cdot x_1(n) \longrightarrow y_1(n) = a x_1^2(n) \quad b \cdot x_2(n) \longrightarrow y_2(n) = b \cdot x_2^2(n)$$

$$y_1(n) + y_2(n) = a x_1^2(n) + b \cdot x_2^2(n) \rightarrow \textcircled{1}$$

$$x(n) = a x_1(n) + b \cdot x_2(n)$$

$$x(n) \longrightarrow \boxed{\text{slm}} \longrightarrow y(n) = x^2(n) \\ = [a x_1(n) + b \cdot x_2(n)]^2 \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2} \quad \therefore \text{The slm is non-linear.}$$

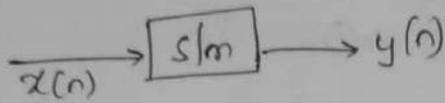
$$h) y(n) = Ax(n) + B.$$

$$a \cdot x_1(n) \longrightarrow aAx_1(n) + B \quad b \cdot x_2(n) \longrightarrow Abx_2(n) + B$$

$$y_1(n) + y_2(n) = Aax_1(n) + B + Abx_2(n) + B$$

$$= A[ax_1(n) + bx_2(n)] + 2B \rightarrow \textcircled{1}$$

$$x(n) = ax_1(n) + bx_2(n)$$



$$y(n) = A \{ax_1(n) + bx_2(n)\} + B \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2}$$

\therefore The given s/m is not linear.

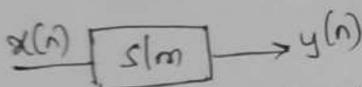
$$\textcircled{1} \ y(n) = e^{x(n)}$$

$$a x_1(n) \rightarrow y_1(n) = a e^{x_1(n)}$$

$$b x_2(n) \rightarrow y_2(n) = b e^{x_2(n)}$$

$$y_1(n) + y_2(n) = a e^{x_1(n)} + b e^{x_2(n)} \rightarrow \textcircled{1}$$

$$x(n) = ax_1(n) + bx_2(n)$$



$$y(n) = e^{ax_1(n) + bx_2(n)} \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2}$$

\therefore The given s/m is non-linear.

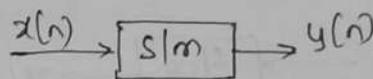
$$\textcircled{1} \ y(n) = x(n) + C$$

$$a x_1(n) \rightarrow y_1(n) = x_1(n) + C$$

$$b x_2(n) \rightarrow y_2(n) = x_2(n) + C$$

$$y_1(n) + y_2(n) = x_1(n) + x_2(n) + 2C \rightarrow \textcircled{1}$$

$$x(n) = ax_1(n) + bx_2(n)$$



$$y(n) = ax_1(n) + bx_2(n) + C \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2}$$

\therefore The given s/m is non-linear.

III. Determine whether the following systems are Causal & Non-causal.

1) $y(n) = x(n) + \frac{1}{x(n-1)}$: causal

2) $y(n) = x(n^2)$: causal

3) $y(n) = x(n) - x(n-1)$: causal

4) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$: causal

5) $y(n) = A \cdot x(n)$: causal

6) $y(n) = x(n) + 3x(n+4)$: Non-causal

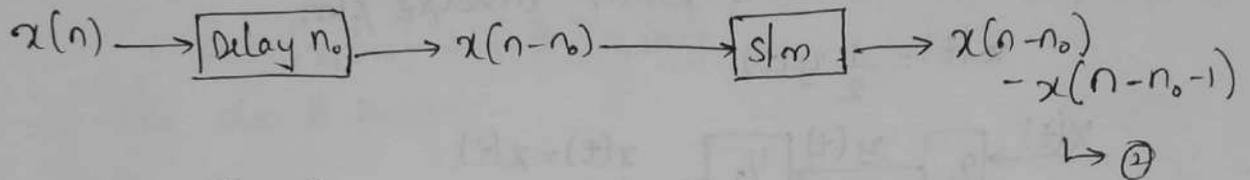
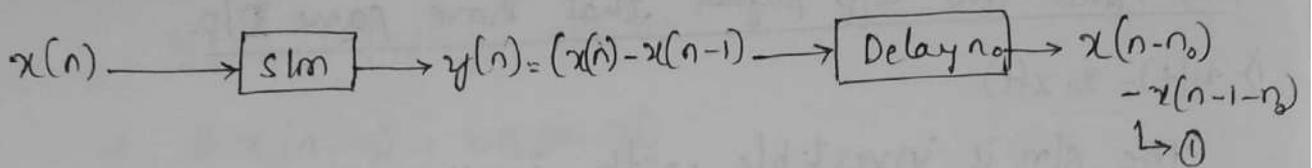
7) $y(n) = x(2n)$: causal

8) $y(n) = \cos x(n)$: causal

9) $y(n) = \sum_{k=-\infty}^1 x(n-k)$: causal

IV Test whether the following systems are time invariant

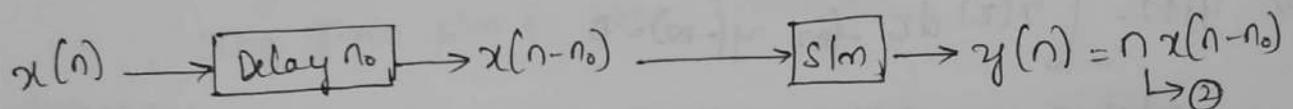
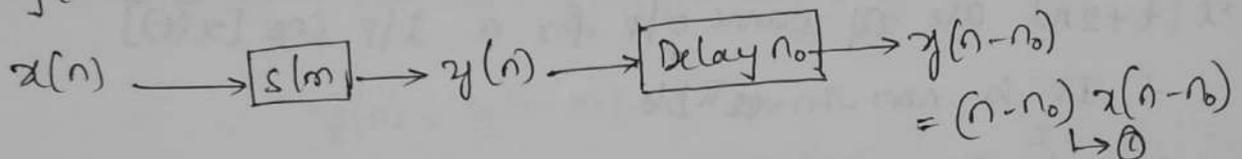
1) $y(n) = x(n) - x(n-1)$



$\textcircled{1} = \textcircled{2}$

\therefore The given slm is time invariant

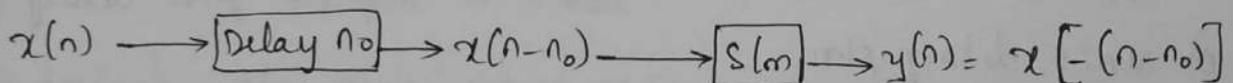
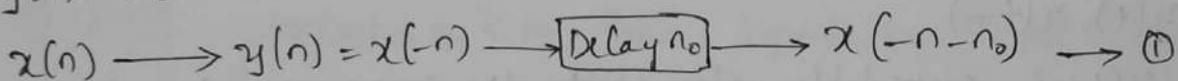
2) $y(n) = n x(n)$



$\textcircled{1} \neq \textcircled{2}$

\therefore The given slm is time variant

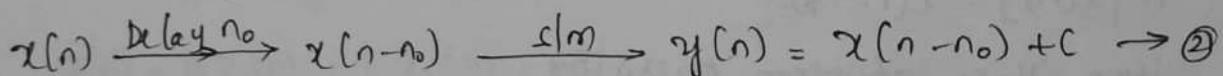
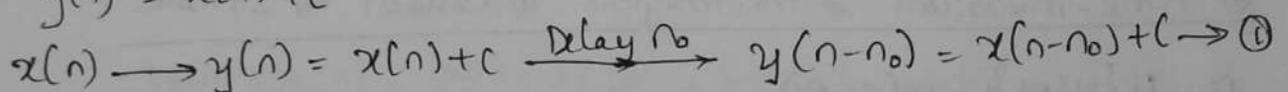
3) $y(n) = x(-n)$



$\textcircled{1} \neq \textcircled{2}$ i.e. $y(n) \neq y(n-n_0) = x(-n+n_0) \rightarrow \textcircled{2}$

\therefore The given slm is time variant

4) $y(n) = x(n) + c$



$y(n) = y(n-n_0) \quad \textcircled{1} = \textcircled{2}$

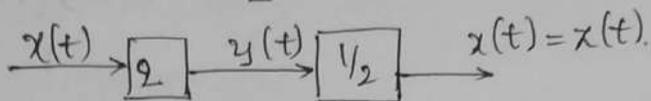
\therefore The given slm is time invariant

V Determine which of the following systems are invertible
 If it is invertible, construct the inverse system. If it is not, find the i/p signal that have same o/p.

1) $y(t) = 2 \cdot x(t)$

The system is invertible with inverse system

$$x(t) = \frac{1}{2} y(t)$$



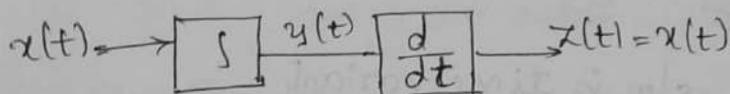
2) $y(t) = \cos[x(t)]$

$y(t) = \cos[x(t)]$ is non-invertible because, $x(t)$ and $x(t+2\pi)$ are of same o/p for a i/p $\cos[x(t)]$

\therefore It is non-invertible.

3) $y(t) = \int_{-\infty}^t x(\tau) d\tau$ for $y(-\infty) = 0$

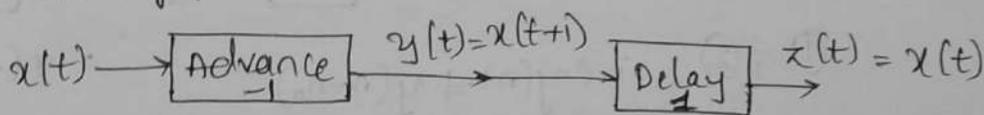
$y(t)$ is invertible with inverse system Differentiator.



$$x(t) = \frac{d}{dt} \left[\int x(\tau) \cdot d\tau \right] = x(t)$$

4) $y(t) = x(t+1)$

$y(t)$ is invertible system with the inverse system delay function & unit delay system.



VI Determine whether the following systems are 1) Linear
 2) Time-invariant 3) memoryless 4) Causal 5) stable.

1) $H\{x(n)\} = x(n-nd)$

Sol: Here 'H' is a system operator.

i.e. $y[n] = H\{x(n)\} = x(n-nd)$

i) Linearity: Let $x(n) = ax_1(n) + bx_2(n)$

Then, $H\{ax_1(n) + bx_2(n)\} = aH\{x_1(n)\} + bH\{x_2(n)\}$

$$y[n] = H\{x(n)\} = x(n - n_d)$$

$$\Rightarrow ax_1(n - n_d) + bx_2(n - n_d)$$

$$= aH\{x_1(n)\} + bH\{x_2(n)\}$$

\therefore The slm is linear.

ii) Time-invariance:

$$H\{x(n)\} = y(n) = x(n - n_d) \rightarrow \text{Delay} \rightarrow y(n - n_0) = x(n - n_d - n_0)$$

$$x(n) \rightarrow \text{delay} \rightarrow x(n - n_0) \rightarrow H\{x(n - n_0)\} = y(n) = x[(n - n_d) - n_0]$$

$$\therefore y(n) = y(n - n_0)$$

\therefore The given slm is Time-invariant.

iii) Memory:

The slm is memoryless unless $n_d = 0$.

iv) Causality: If $n_d \geq 0$, the slm is causal otherwise it is not causal.

v) Stability: $|x(n)| \leq B_x < \infty$ then, $|y(n)| = |x(n - n_d)| \leq B_x < \infty$
 \therefore the slm is stable

2) $T\{x(n)\} = g(n) \cdot x(n)$, where T is a slm operator

Soln:

$$i) \text{ Linearity: } ax_1(n) \xrightarrow[\text{slm } T\{\cdot\}]{\text{slm}} g(n) \cdot ax_1(n) = y_1(n)$$

$$bx_2(n) \xrightarrow[\text{slm } T\{\cdot\}]{\text{slm}} g(n) \cdot bx_2(n) = y_2(n)$$

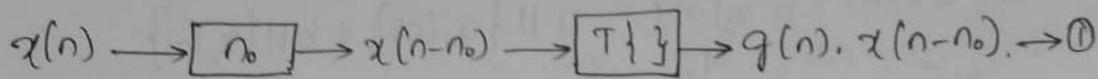
$$y_1(n) + y_2(n) = g(n) \{ax_1(n) + bx_2(n)\} \rightarrow \textcircled{1}$$

$$x(n) = ax_1(n) + bx_2(n)$$

$$\therefore T\{x(n)\} = g(n) \{ax_1(n) + bx_2(n)\} \rightarrow \textcircled{2}$$

$\textcircled{1} = \textcircled{2} \Rightarrow$ The given slm is linear.

i) Time invariance: $T\{x(n)\} = g(n)x(n)$.



$$y(n) = g(n) \cdot x(n)$$

$$y(n-n_0) = g(n-n_0) \cdot x(n-n_0) \rightarrow \textcircled{2}$$

$\textcircled{1} \neq \textcircled{2} \therefore$ The given system is timevariant.

iii) Memory: Since the value of the o/p depends only on the present value of the input. The given system is memoryless.

iv) Causality: Since the given system o/p depends only on the present value of the i/p. The given system is causal.

v) Stability: Let $|x(n)| \leq B_x$

$$\begin{aligned} \text{then } |y(n)| &= |g(n) \cdot x(n)| = |g(n)| \cdot |x(n)| \\ &= |g(n)| B_x. \end{aligned}$$

The o/p of the system is bounded only if $g(n)$ is bounded.

3) $T\{x(n)\} = x(-n)$, T is a system operator

Soln: $y(n) = T\{x(n)\} = x(-n)$.

i) Linearity: $a x_1(n) \rightarrow y_1(n) = a x_1(-n)$

$$b x_2(n) \rightarrow y_2(n) = b x_2(-n)$$

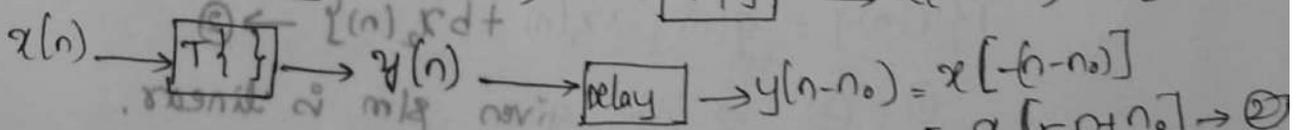
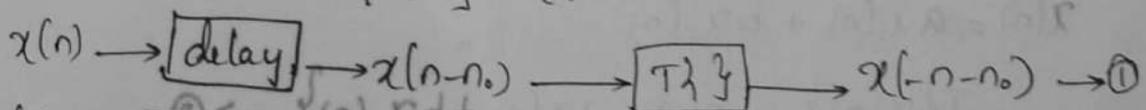
$$y_1(n) + y_2(n) = a x_1(-n) + b x_2(-n) \rightarrow \textcircled{1}$$

$$\text{Let } x(n) = a x_1(n) + b x_2(n)$$

$$\begin{aligned} x(n) &\rightarrow T\{a x_1(n) + b x_2(n)\} = a T\{x_1(n)\} + b T\{x_2(n)\} \\ &= a x_1(-n) + b x_2(-n) \rightarrow \textcircled{2} \end{aligned}$$

$\textcircled{1} = \textcircled{2} \therefore$ The given system is linear.

ii) Time invariance: $T\{x(n)\} = x(-n)$



$$\textcircled{1} \neq \textcircled{2} \text{ i.e. } y(n-n_0) \neq y(n)$$

\therefore The given s/m is Time Variant.

iii) memory: If $n > 0$, the s/m is not memoryless.

iv) causality: $T\{x(n)\} = x(-n)$

$$\text{let } n = -5, \text{ then } T\{x(-5)\} = x(5)$$

\therefore The value of the o/p signal at $n = -5$ depends on the value of i/p at $n = 5$. Therefore the o/p depends on the future values of the i/p. \therefore s/m is non-causal.

v) stability: If $|x(n)| \leq B_x$ then, $|y(n)| = |x(-n)| \leq B_x$

\therefore The s/m is stable.

4) $T\{x(n)\} = x(n) + u(n+1)$

$$y(n) = T\{x(n)\} = x(n) + u(n+1)$$

i) Linearity: $a x_1(n) \rightarrow T\{ \cdot \} \rightarrow a x_1(n) + u(n+1) = y_1(n)$

$$b x_2(n) \rightarrow T\{ \cdot \} \rightarrow b x_2(n) + u(n+1) = y_2(n)$$

$$y_1(n) + y_2(n) = a x_1(n) + u(n+1) + b x_2(n) + u(n+1)$$

$$= a x_1(n) + b x_2(n) + 2u(n+1) \rightarrow \textcircled{1}$$

$$x(n) = a x_1(n) + b x_2(n)$$

$$\therefore T\{x(n)\} = T\{a x_1(n) + b x_2(n)\}$$

$$= a x_1(n) + b x_2(n) + u(n+1) \rightarrow \textcircled{2}$$

$\textcircled{1} \neq \textcircled{2} \therefore$ The given s/m is not linear.

ii) Time invariance:

$$x(n) \rightarrow \text{Delay } n_0 \rightarrow x(n-n_0) \rightarrow T\{ \cdot \} \rightarrow x(n-n_0) + u(n+1)$$

$\hookrightarrow \textcircled{1}$

$$x(n) \rightarrow T\{ \cdot \} \rightarrow y(n) \rightarrow \text{Delay } n_0 \rightarrow y(n-n_0) = x(n-n_0) + u(n-n_0+1)$$

$\hookrightarrow \textcircled{2}$

$$\textcircled{1} \neq \textcircled{2} \text{ i.e. } y(n-n_0) \neq T\{x(n-n_0)\} \quad \textcircled{2} = \textcircled{1}$$

\therefore The given s/m is not time invariant.

iii) memory: The o/p of the s/m, $y(n)$ depends only on the present values of the input \therefore It is memoryless

iv) Causal: The o/p of the s/m depends only on the present values & It is independent of future values of i/p.

\therefore The given s/m is causal s/m.

v) Stability: If $|x(n)| \leq B_x$ then

$$\begin{aligned} |y(n)| &= |x(n) + u(n+1)| \\ &\leq |x(n)| + |u(n+1)| \\ &\leq B_x + |u(n+1)| \end{aligned}$$

\therefore The s/m is stable.

5) $y(t) = e^{x(t)}$

Soln: $y(t) = T\{x(t)\} = e^{x(t)}$

i) Linearity: $T\{ax_1(t)\} \rightarrow \boxed{T\{ \cdot \}} \rightarrow y_1(t) = e^{ax_1(t)}$

$bx_2(t) \rightarrow \boxed{T\{ \cdot \}} \rightarrow y_2(t) = e^{bx_2(t)}$

$y_1(t) + y_2(t) = e^{ax_1(t)} + e^{bx_2(t)} \rightarrow \textcircled{1}$

Let $x(t) = ax_1(t) + bx_2(t)$

$T\{x(t)\} = T\{ax_1(t) + bx_2(t)\}$

$= e^{ax_1(t) + bx_2(t)} = e^{ax_1(t)} \cdot e^{bx_2(t)} \rightarrow \textcircled{2}$

$\textcircled{1} \neq \textcircled{2}$

ie $T\{ax_1(t) + bx_2(t)\} \neq T\{ax_1(t)\} + T\{bx_2(t)\}$

\therefore The given s/m is non-linear.

ii) Time invariance:

$x(t) \rightarrow \boxed{\text{Delay } t_0} \rightarrow x(t-t_0) \rightarrow \boxed{T\{ \cdot \}} \rightarrow e^{x(t-t_0)} \rightarrow \textcircled{1}$

$x(t) \rightarrow \boxed{T\{ \cdot \}} \rightarrow y(t) \rightarrow \boxed{\text{Delay}} \rightarrow e^{x(t-t_0)} \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ i.e. $y(t) = T\{x(t-t_0)\} = y(t-t_0)$

\therefore The given s/m is time invariant.

iii) memory: $y(t) = e^{x(t)}$

The o/p of the s/m depends only on the present values of

i/p \therefore The given s/m is memoryless.

iv) Causality: The given s/m's o/p is independent of future values of i/p. \therefore The s/m is causal.

v) stability: Let $|x(t)| \leq B_x$

$$\text{then } |y(t)| = |e^{x(t)}| \leq B_x$$

If the i/p is bounded, then the o/p is also bounded. Therefore the s/m is stable.

6) $y(t) = \frac{dx(t)}{dt}$

Soln: $y(t) = T\{x(t)\} = \frac{dx(t)}{dt}$

Linearity: $a x_1(t) \longrightarrow a \cdot \frac{dx_1(t)}{dt} = y_1(t)$

$b x_2(t) \longrightarrow b \cdot \frac{dx_2(t)}{dt} = y_2(t)$

$y_1(t) + y_2(t) = a \cdot \frac{dx_1(t)}{dt} + b \cdot \frac{dx_2(t)}{dt} \rightarrow \textcircled{1}$

Let $x(t) = a x_1(t) + b x_2(t)$

$T\{x(t)\} = T\{a x_1(t) + b x_2(t)\}$

$= \frac{d}{dt} [a x_1(t) + b x_2(t)] = a \cdot \frac{dx_1(t)}{dt} + b \cdot \frac{dx_2(t)}{dt} \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ i.e. $T\{a x_1(t) + b x_2(t)\} = T\{a x_1(t)\} + T\{b x_2(t)\}$

\therefore The given s/m is Linear.

Time Invariance:

$x(t) \longrightarrow \text{Delay} \longrightarrow x(t-t_0) \longrightarrow T\{y\} \longrightarrow \frac{dx(t-t_0)}{dt}$

$x(t) \longrightarrow T\{y\} \longrightarrow y(t) \longrightarrow \text{Delay} \longrightarrow y(t-t_0) = \frac{dx(t-t_0)}{dt}$

$\therefore T\{x(t-t_0)\} = y(t-t_0) \therefore$ The given s/m is time invariant

iii) memory: The o/p depends on the present value. But differentiator has a memory.

iv) Causal: The o/p does not depend on the future values of the i/p. So the s/m is causal.

v) Stability: If $|x(t)| \leq B_x$ then $|y(t)| = \left| \frac{dx(t)}{dt} \right| \neq B_y$
 \therefore The s/m is unstable ex: if $x(t) = u(t) = \frac{d u(t)}{dt} = x(t)$

$\Rightarrow y(t) = x(t/2)$

Soln: $y(t) = T\{x(t)\} = x(t/2)$

Linearity: $a x_1(t) \rightarrow [T\{\}] \rightarrow y_1(t) = a x_1(t/2)$

$b x_2(t) \rightarrow [T\{\}] \rightarrow y_2(t) = b x_2(t/2)$

$y_1(t) + y_2(t) = a x_1(t/2) + b x_2(t/2) \rightarrow \textcircled{1}$

Let $x(t) = a x_1(t) + b x_2(t)$

$T\{x(t)\} = T\{a x_1(t) + b x_2(t)\}$
 $= a x_1(t/2) + b x_2(t/2) \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$
 $T\{a x_1(t) + b x_2(t)\} = T\{a x_1(t)\} + T\{b x_2(t)\}$

\therefore The s/m is linear.

ii) Time Invariance:

$x(t) \rightarrow \text{Delay } t_0 \rightarrow x(t-t_0) \rightarrow [T\{\}] \rightarrow x\left(\frac{t-t_0}{2}\right)$

$x(t) \rightarrow [T\{\}] \rightarrow y(t) \rightarrow \text{Delay } t_0 \rightarrow y(t-t_0) = x\left(\frac{t-t_0}{2}\right)$

$y(t-t_0) \neq T\{x(t-t_0)\}$

\therefore The given s/m is not time invariant.

iii) memory: The o/p depends on the past values of the i/p.

\therefore The s/m is not memoryless. It has a memory.

iv) Causal: The s/m is not independent of future values of i/p

i.e. $y(-1) = x(-1/2) = x(-0.5)$. \therefore The s/m is non-causal.

\checkmark Stability: Let $|x(t)| \leq B_x$
 then $|y(t)| = |x(t)| \leq B_x$
 \therefore system is stable.

$\Rightarrow y(t) = \cos(x(t))$

Soln: $y(t) = T\{x(t)\} = \cos(x(t))$

\Rightarrow Linearity: $a x_1(t) \xrightarrow{\{ \}} y_1(t) = a \cos(x_1(t))$

$b x_2(t) \xrightarrow{\{ \}} y_2(t) = b \cos(x_2(t))$

$y_1(t) + y_2(t) = a \cos(x_1(t)) + b \cos(x_2(t)) \rightarrow \textcircled{1}$

Let $x(t) = a x_1(t) + b x_2(t)$

$T\{x(t)\} = T\{a x_1(t) + b x_2(t)\}$
 $= \cos\{a x_1(t) + b x_2(t)\} \rightarrow \textcircled{2}$

$\textcircled{1} \neq \textcircled{2}$ i.e. $T\{a x_1(t)\} + T\{b x_2(t)\} \neq T\{a x_1(t) + b x_2(t)\}$

\therefore The s/m is non-linear.

ii) Time invariance:

$x(t) \xrightarrow{\text{Delay } t_0} x(t-t_0) \xrightarrow{T\{ \}} y(t) = \cos(x(t-t_0)) \rightarrow \textcircled{1}$

$x(t) \xrightarrow{\{ \}} y(t) \xrightarrow{\text{Delay}} y(t-t_0) = \cos(x(t-t_0)) \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ i.e. $y(t-t_0) = T\{x(t-t_0)\}$

\therefore The given s/m is time invariant.

iii) memory: The output depends only on the present values of the i/p. So the s/m is memoryless.

iv) Causal: The o/p of the s/m does not depend on the future values of the i/p. So the s/m is causal.

\checkmark Stability: Let $|x(t)| \leq B_x$
 $|y(t)| = |\cos(x(t))| \leq 1 < \infty$

\therefore The s/m is stable.

$$q) y(t) = \frac{d}{dt} \{ e^{-t} x(t) \}$$

soln: $y(t) = T\{x(t)\} = \frac{d}{dt} \{ e^{-t} x(t) \}$

i) Linearity: $ax_1(t) \longrightarrow y_1(t) = \frac{d}{dt} \{ e^{-t} ax_1(t) \}$

$$bx_2(t) \longrightarrow y_2(t) = \frac{d}{dt} \{ e^{-t} bx_2(t) \}$$

$$y_1(t) + y_2(t) = \frac{d}{dt} \{ e^{-t} ax_1(t) \} + \frac{d}{dt} \{ e^{-t} bx_2(t) \} \rightarrow \textcircled{1}$$

Let $x(t) = ax_1(t) + bx_2(t)$

$$T\{x(t)\} = T\{ax_1(t) + bx_2(t)\}$$

$$= \frac{d}{dt} \{ e^{-t} (ax_1(t) + bx_2(t)) \} = \frac{d}{dt} \{ e^{-t} ax_1(t) \} + \frac{d}{dt} \{ e^{-t} bx_2(t) \} \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2} \text{ i.e. } T\{ax_1(t) + bx_2(t)\} = T\{ax_1(t)\} + T\{bx_2(t)\}$$

\therefore The s/m is Linear

ii) Time invariance:

$$x(t) \xrightarrow{\text{delay to } t_0} x(t-t_0) \xrightarrow{T\{\cdot\}} y(t) = \frac{d}{dt} \{ e^{-t} x(t-t_0) \} \rightarrow \textcircled{1}$$

$$x(t) \xrightarrow{T\{\cdot\}} y(t) \xrightarrow{\text{Delay to } t_0} y(t-t_0) = \frac{d}{dt} \{ e^{-(t-t_0)} x(t-t_0) \} \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2} \text{ i.e. } y(t-t_0) \neq T\{x(t-t_0)\}$$

\therefore The given s/m is not time invariant

iii) memory: The s/m has a memory

iv) causal: The op of the s/m is independent of future values of i/p. Therefore the s/m is causal.

v) stability: Let $|x(t)| \leq B_x$

$$|y(t)| = \left| \frac{d}{dt} e^{-t} x(t) \right| \leq B_y$$

Therefore the s/m is stable.

$$10) y(n) = 2 \cdot x(n) \cdot u(n).$$

Soln: $y(n) = T\{x(n)\} = 2 \cdot x(n) \cdot u(n)$

i) Linearity: $ax_1(n) \longrightarrow y_1(n) = 2ax_1(n) \cdot u(n)$

$$bx_2(n) \longrightarrow y_2(n) = 2bx_2(n) \cdot u(n)$$

$$y_1(n) + y_2(n) = 2u(n) \{ax_1(n) + bx_2(n)\} \longrightarrow \textcircled{1}$$

$$x(n) = ax_1(n) + bx_2(n)$$

$$T\{x(n)\} = T\{ax_1(n) + bx_2(n)\}$$

$$= 2 \cdot u(n) \cdot \{ax_1(n) + bx_2(n)\} \longrightarrow \textcircled{2}$$

$\textcircled{1} = \textcircled{2} \therefore$ The slm is linear

ii) Time invariance:

$$x(t) \longrightarrow \boxed{\text{Delay } t_0} \longrightarrow x(t-t_0) \longrightarrow \boxed{\text{slm}} \longrightarrow y(t) = 2 \cdot x(t-t_0) \cdot u(t) \quad \longmapsto \textcircled{1}$$

$$x(n) \longrightarrow y(n) \longrightarrow \boxed{\text{Delay } n_0} \longrightarrow y(n-n_0) = 2 \cdot x(n-n_0) \cdot u(n-n_0) \quad \longmapsto \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2} \quad \text{i.e. } y(t-t_0) \neq T\{x(t-t_0)\}$$

\therefore The given slm is time variant

iii) Memory: slm is memoryless. since the o/p depends only the present values of the i/p.

iv) Causal: The o/p of the slm is independent of the future values of i/p. Therefore it is causal.

v) Stability: for $|x(n)| \leq B_x$

$$\text{then } |y(n)| = |2x(n) \cdot u(n)| \leq B_x$$

\therefore system is stable.

$$11) y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

Soln: $y(n) = T\{x(n)\} = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

Here $x(n)$ is only i/p
 $\delta(n)$ is constant.

i) Linearity: $a x_1(n) \longrightarrow y_1(n) = a x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$b x_2(n) \longrightarrow y_2(n) = b x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$$y_1(n) + y_2(n) = a x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) + b x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$= \sum_{k=-\infty}^{\infty} \delta(n-2k) \{ a x_1(n) + b x_2(n) \} \longrightarrow \textcircled{1}$$

Let $x(n) = a x_1(n) + b x_2(n)$

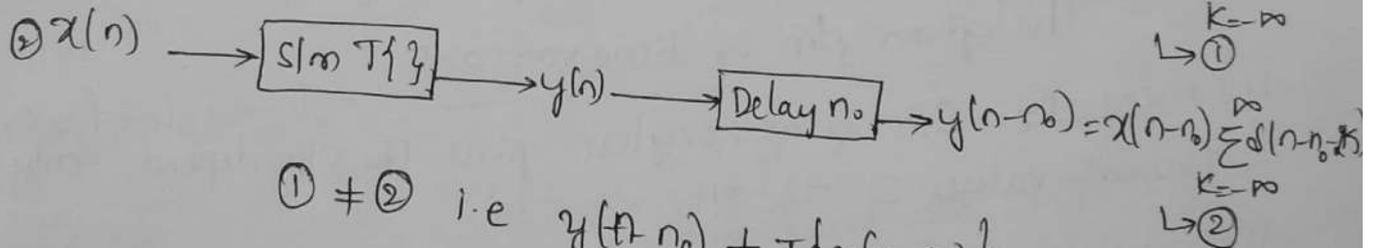
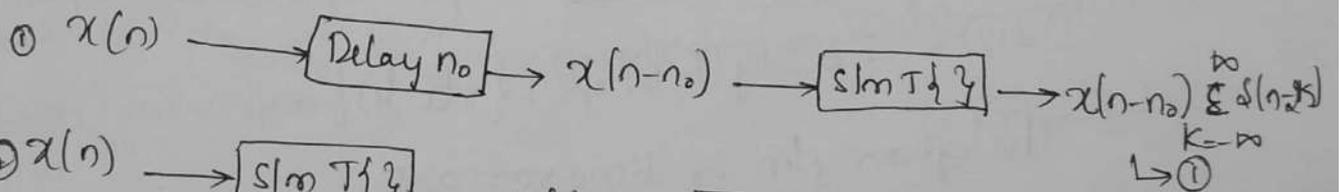
$$T \{ x(n) \} = T \{ a x_1(n) + b x_2(n) \}$$

$$= \{ a x_1(n) + b x_2(n) \} \sum_{k=-\infty}^{\infty} \delta(n-2k) \longrightarrow \textcircled{2}$$

$\textcircled{1} = \textcircled{2}$ i.e. $T \{ a x_1(n) + b x_2(n) \} = T \{ a x_1(n) \} + T \{ b x_2(n) \}$

\therefore The s/m is linear

ii) Time invariance:



$\textcircled{1} \neq \textcircled{2}$ i.e. $y(n-n_0) \neq T \{ x(n-n_0) \}$

\therefore The given s/m is not time invariant.

iii) memory: It is memoryless

iv) Causal: The s/m is causal because it is independent of future values of i/p.

v) stability: Let $|x(n)| \leq B_x$

then $|y(n)| = |x(n)| \sum_{k=-\infty}^{\infty} \delta(n-2k) \leq B_y$

\therefore The s/m is stable.

12) $y(n) = \log_{10}(|x(n)|)$

Soln: $y(n) = T\{x(n)\} = \log_{10}(|x(n)|)$

i) Linearity: $ax_1(n) \rightarrow y_1(n) = \log_{10}(|ax_1(n)|)$

$bx_2(n) \rightarrow y_2(n) = \log_{10}(|bx_2(n)|)$

$y_1(n) + y_2(n) = \log_{10}(|ax_1(n)|) + \log_{10}(|bx_2(n)|) \rightarrow \textcircled{1}$

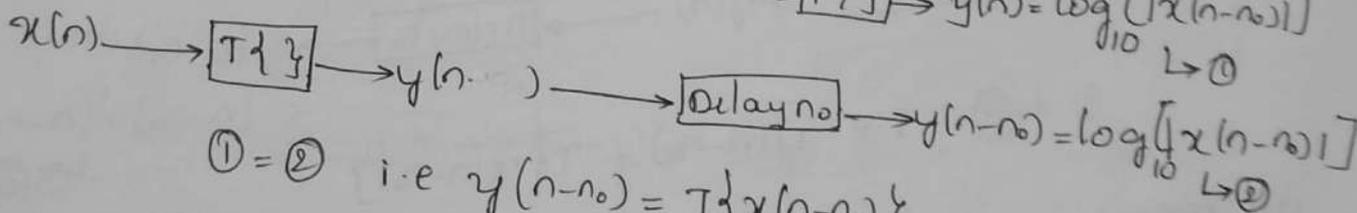
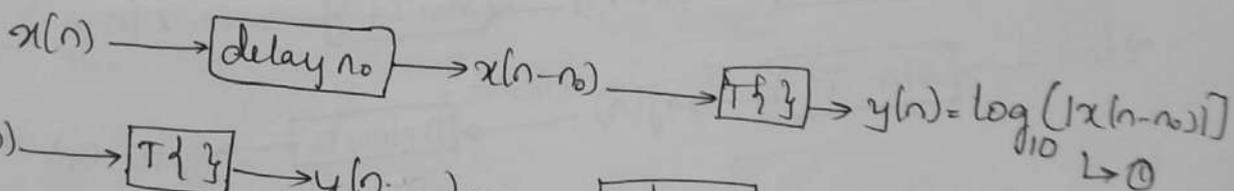
$x(n) = \{ax_1(n) + bx_2(n)\}$

$T\{x(n)\} = T\{ax_1(n) + bx_2(n)\}$
 $= \log_{10}(|ax_1(n) + bx_2(n)|) \rightarrow \textcircled{2}$

$\textcircled{1} \neq \textcircled{2}$ i.e. $T\{ax_1(n) + bx_2(n)\} \neq T\{x_1(n)\} + T\{x_2(n)\}$

\therefore The slm is non-linear

ii) Time invariance:



$\textcircled{1} = \textcircled{2}$ i.e. $y(n-n_0) = T\{x(n-n_0)\}$

\therefore The slm is time invariant.

iii) memory: O/p depends only on the present values of i/p.

\therefore It is memoryless.

iv) Causal: The o/p depends on the present values and is independent of future values of i/p. It is causal slm

v) Stability: Let $|x(n)| \leq B_x$

then $|y(n)| = |\log_{10}(|x(n)|)| \leq B_x$

\therefore system is stable

$$13) \quad y(n) = n \cdot x(n)$$

Soln: $y(n) = T\{x(n)\} = n \cdot x(n)$

i) Linearity: $ax_1(n) \rightarrow y_1(n) = a \cdot n \cdot x_1(n)$

$$bx_2(n) \rightarrow y_2(n) = n \cdot b x_2(n)$$

$$y_1(n) + y_2(n) = a \cdot n x_1(n) + b n x_2(n) = n \{ a x_1(n) + b x_2(n) \} \rightarrow \textcircled{1}$$

Let $x(n) = \{ a x_1(n) + b x_2(n) \}$

$$T\{x(n)\} = T\{ a x_1(n) + b x_2(n) \}$$

$$= n \{ a x_1(n) + b x_2(n) \} \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2} \quad \text{i.e. } T\{ a x_1(n) + b x_2(n) \} = T\{ a x_1(n) \} + T\{ b x_2(n) \}$$

\therefore The s/m is linear

ii) Time invariance:

$$x(n) \rightarrow \boxed{\text{Delay } n_0} \rightarrow x(n-n_0) \rightarrow \boxed{\text{slm } T\{ \cdot \}} \rightarrow n \cdot x(n-n_0) \rightarrow \textcircled{1}$$

$$x(n) \rightarrow \boxed{\text{slm } T\{ \cdot \}} \rightarrow y(n) \rightarrow \boxed{\text{Delay } n_0} \rightarrow y(n-n_0) = (n-n_0) x(n-n_0) \rightarrow \textcircled{2}$$

$$\textcircled{1} \neq \textcircled{2} \quad \text{i.e. } y(n-n_0) \neq T\{ x(n-n_0) \}$$

iii) Memory: the o/p depends only on the present values of the i/p. It is memoryless

iv) Causal: The o/p of slm is independent of future values of i/p. Therefore it is causal s/m

v) Stability: Let $|x(n)| \leq B_x$

$$|y(n)| = |n \cdot x(n)| \leq B_x$$

Since 'n' is not bounded, s/m is unstable

(14) $y(n) = \sum_{k=-\infty}^{\infty} x(k+2)$

Soln: $y(n) = T\{x(n)\} = \sum_{k=-\infty}^{\infty} x(k+2)$

i) Linearity: $a x_1(n) \rightarrow y_1(n) = \sum_{k=-\infty}^{\infty} a \cdot x_1(k+2)$

$b x_2(n) \rightarrow y_2(n) = \sum_{k=-\infty}^{\infty} b \cdot x_2(k+2)$

$y_1(n) + y_2(n) = a \sum_{k=-\infty}^{\infty} x_1(k+2) + b \sum_{k=-\infty}^{\infty} x_2(k+2) \rightarrow \textcircled{1}$

$x(n) = \{a x_1(n) + b x_2(n)\}$

$T\{x(n)\} = T\{a x_1(n) + b x_2(n)\}$

$y = \sum_{k=-\infty}^{\infty} a x_1(k+2) + b x_2(k+2) \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ i.e. $T\{a x_1(n) + b x_2(n)\} = T\{a x_1(n)\} + T\{b x_2(n)\}$

\therefore The s/m is Linear

ii) Time Invariance:

$x(n) \rightarrow \text{Delay } n_0 \rightarrow x(n-n_0) \rightarrow \text{s/m} \rightarrow \sum_{k=-\infty}^{\infty} x(k+2-n_0) \rightarrow \textcircled{1}$

$x(n) \rightarrow \text{s/m} \rightarrow y(n) \rightarrow \text{Delay } n_0 \rightarrow y(n-n_0) = \sum_{k=-\infty}^{\infty} x(k+2-n_0) \rightarrow \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ i.e. $y(n-n_0) = T\{x(n-n_0)\}$

\therefore The s/m is time invariant

iii) memory: $y(n) = \sum_{k=-\infty}^{\infty} x(k+2)$

\checkmark & Causal

for $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k+2) = x(-\infty) + \dots + x(-1) + x(0) + x(1) + x(2) + x(3) + \dots$

The s/m o/p depends on the past and future values of i/p. Therefore it is not memoryless & it is non-causal.

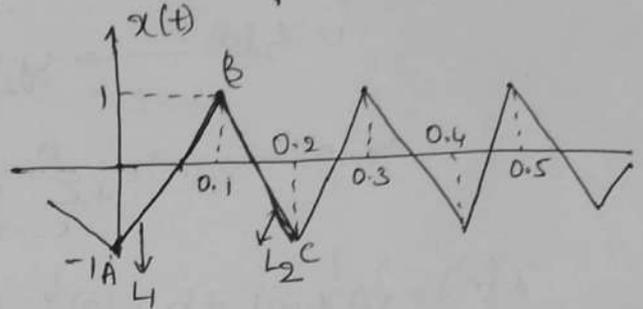
✓ Stability: let $|x(n)| \leq B_x$

then $|y(n)| = \left| \sum_{k=-\infty}^{\infty} x(k+2) \right| \leq B_y$

∴ System is stable.

15) What is the average power of the triangular wave shown in fig.

Soln: It is periodic with $T=0.2$
 L_1 & L_2 are repeating forming triangular wave.



Average power $P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \cdot dt$

$L_1 = AB$ $L_2 = BC$
 $A = (x_1, y_1) = (0, -1)$ $B = (x_2, y_2) = (0.1, 1)$

AB's slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 + 1}{0.1 + 0}$

$x(t) = 20t - 1 ; 0 < t < 0.1$

$= -20t + 3 ; 0.1 < t < 0.2$

$m = \frac{2}{0.1} = 20$

$P = \frac{1}{0.2} \left[\int_0^{0.1} (20t - 1)^2 \cdot dt + \int_{0.1}^{0.2} (-20t + 3)^2 \cdot dt \right]$

$(y - y_1) = m(x - x_1)$

$y + 1 = 20(x - 0)$

$y = 20x - 1$

$y \rightarrow x(t) \quad x \rightarrow t$

$x(t) = 20t - 1 \rightarrow \textcircled{1}$

~~$= \frac{1}{0.2} \left[\frac{(20t - 1)^3}{3} \Big|_0^{0.1} + \frac{(-20t + 3)^3}{3} \Big|_{0.1}^{0.2} \right]$~~

$L_2: BC \quad B = (0.1, 1), C = (0.2, -1)$
 $x_1, y_1 \quad x_2, y_2$

BC's slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - 1}{0.2 - 0.1} = -20$

~~$= \frac{1}{0.2} \left[\frac{(1)^3}{3} - \frac{(-1)^3}{3} + \left(\frac{-1}{3} \right) - \left(\frac{1}{3} \right) \right]$~~

$(y - y_1) = m(x - x_1)$

$y - 1 = -20(x - 0.1)$

$y = -20x + 2 + 1$

$y = -20x + 3$

$x(t) = -20t + 3 \rightarrow \textcircled{2}$

~~$= \frac{1}{0.2} \left[\frac{1}{3} + \frac{1}{3} - \frac{1}{3} \right]$~~

~~$\frac{1}{0.2} \left[\frac{1}{3} + \frac{1}{3} - \frac{1}{3} \right]$~~

$P = \frac{1}{3} \text{ watt}$